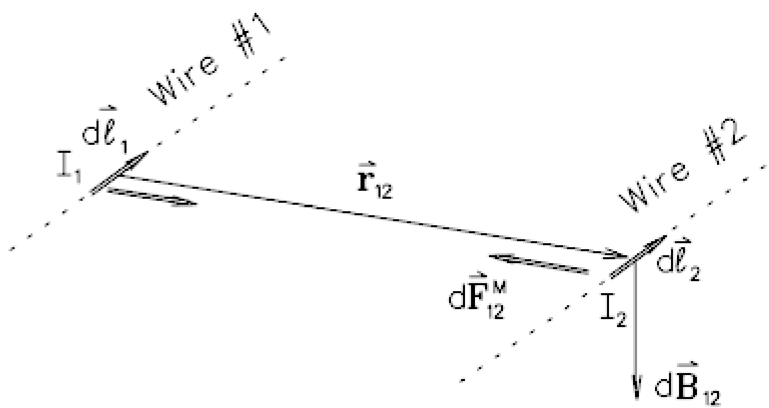
Direct Force Laws

The Coulomb force

$$\vec{F}^E_{12} = k_E \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

of charge q_1 on charge q_2 located at position $\boldsymbol{r}_{12}$ relative to the position of q_1 is already too complicated; instead we write $\boldsymbol{F}_{12} = q_2 \boldsymbol{E}$ and find $\boldsymbol{E}$ from **Coulomb's Law**:

$$\vec{E} = k_E \frac{Q}{r^2} \hat{r}$$



Even more intricate and confusing is the direct force between two current elements:

$$d\vec{F}^M_{12} = k_M \frac{I_1 I_2}{r_{12}^2} d\vec{\ell}_2 \times (d\vec{\ell}_1 \times \hat{r}_{12})$$

For this, we presuppose a magnetic field $\boldsymbol{B}$ at the position of $I_2\, d\boldsymbol{\ell}_2 = q\, \boldsymbol{v}_2$ and combine both types of forces into one general electromagnetic force law:

the **Lorentz Force**:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

The Lorentz Force

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

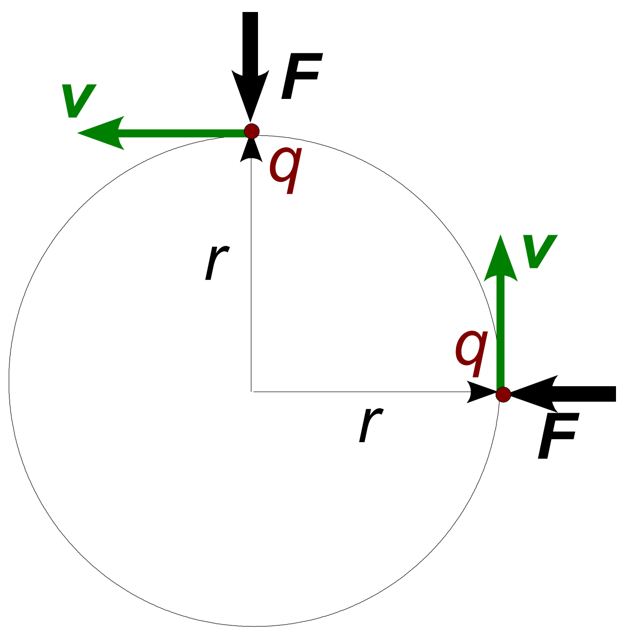
There are lots of applications of the Lorentz force, as you might expect. (After all, **force** is what we need to do some **work**!) We will look at:

- **Circulating Charges**: when $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$ we get a force $\boldsymbol{F}$ that is perpendicular to both. This produces **uniform circular motion.**
  Cyclotrons: $p = qBr$ where p = momentum and r = orbit radius.
  Magnetic Mirrors: Magnetic forces **do no work**. Spiral paths reflect.
- **Velocity Selectors**: when $\boldsymbol{v}$ is perpendicular to both $\boldsymbol{E}$ and $\boldsymbol{B}$ we can adjust the ratio until $E/B = v$ so $F = 0$. If p is known, so is m.
- **Hall Effect**: charges moving down a conductor through a perpendicular magnetic field get swept sideways until a voltage builds up.
- **Rail Guns**: discharge a capacitor to make a huge current pulse....

The Cyclotron

When **v** is perpendicular to **B** we get a force **F** that is perpendicular to both. This is the familiar criterion for **uniform circular motion**. Recall

$mv^2/r = qvB$ or $p = qBr$ where $p = mv$.



Since $v = r\omega$ we have $mr\omega = qBr$

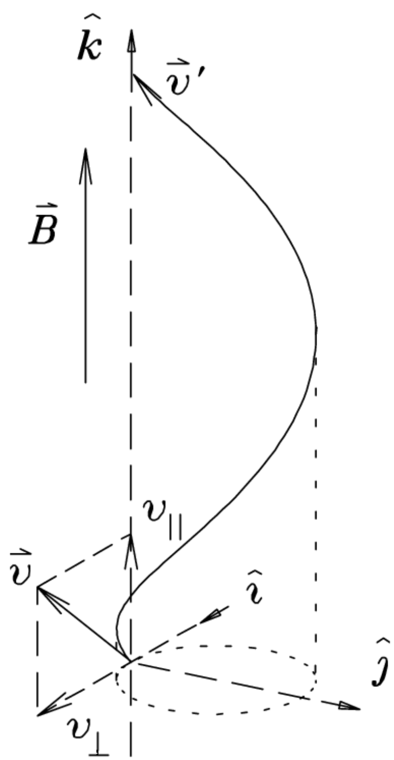
or

$$\omega = qB/m = \text{constant}.$$

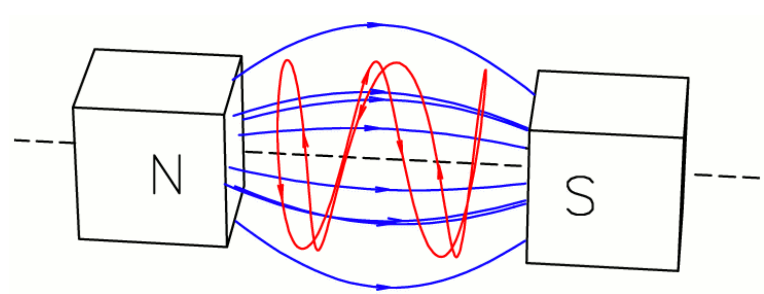
If the frequency of the charged particle orbit is constant, we can apply an accelerating voltage to the particles that reverses direction every half-orbit so that it is always in the right direction to make the particles go faster. This is what we call a **cyclotron**.

Magnetic Mirrors

Circulating Charges: when ***v*** is perpendicular to ***B*** we get a force ***F*** that is perpendicular to both. This produces **uniform circular motion**. This works on $v_\perp$, the **perpendicular component** of ***v***, in the same way. Any component $v_{||}$ of ***v*** that is **parallel** to ***B*** is **unaffected**. The net result is a **spiral** path:



What about **nonuniform** magnetic fields? If $v_{||}$ points into a region of stronger ***B***, then $v_{||}$ can't be parallel on both sides of the orbit, so $v_{||}$ gets smaller and eventually reverses direction. Remember, magnetic forces **do no work**, so this **reflection** is **perfectly elastic**! Many examples:



Aurorae

Tokamaks

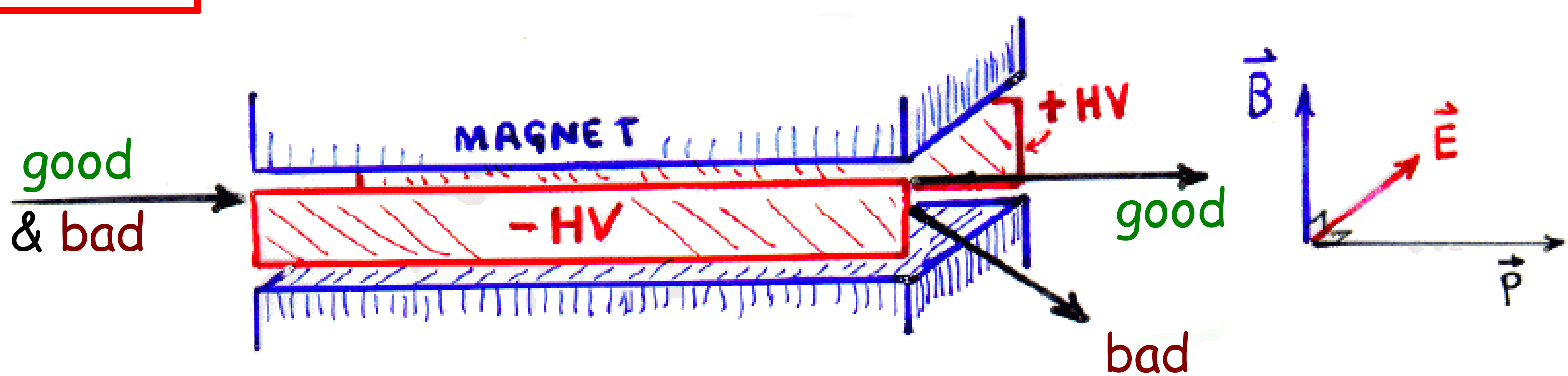
Galactic accelerators

Velocity Selectors

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

When v is perpendicular to both E and B we can adjust the ratio until

$E/B = v$ so that $F = 0$.



If p is known (as it usually is), then so is m. This makes such devices handy as **mass separators**. You will see several at TRIUMF.

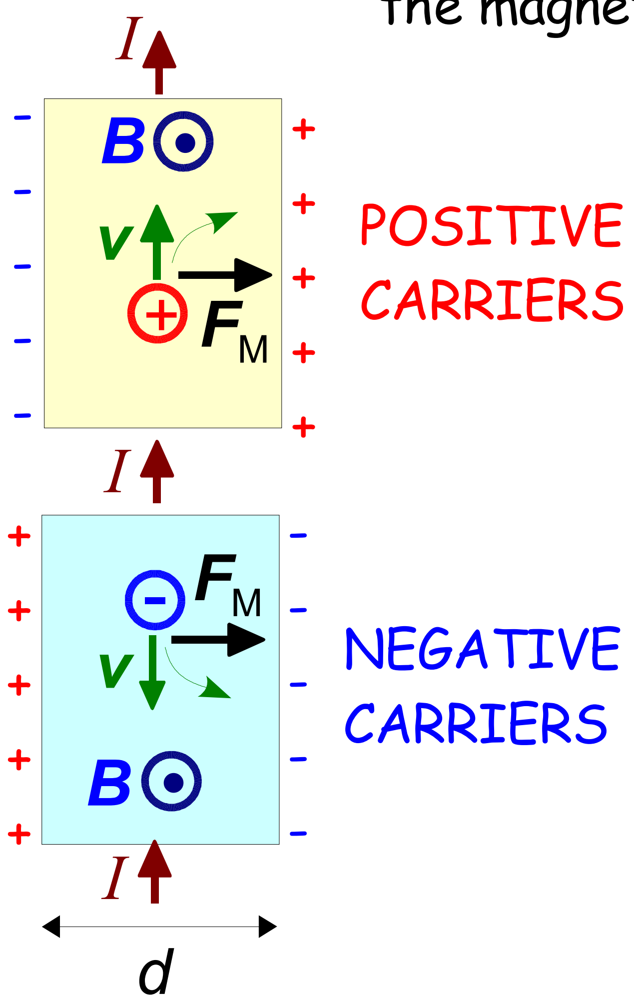
The Hall Effect:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Charges moving down a conductor through a perpendicular magnetic field get swept sideways until an electric field E_{Hall} builds up due to the accumulated surface charges. When qE_{Hall} is just big enough to cancel the magnetic force $F_M = qvB$, the charges are no longer deflected. This implied a Hall field of $E_{Hall} = vB$, giving a **Hall voltage** of

$$V_{Hall} = vBd$$

across a conductor of width d.



Now, the current density is $J = nqv$, where n is the # of carriers per unitvolume, so $v = J/nq$ and thus $V_{Hall} = JBd/nq$

or

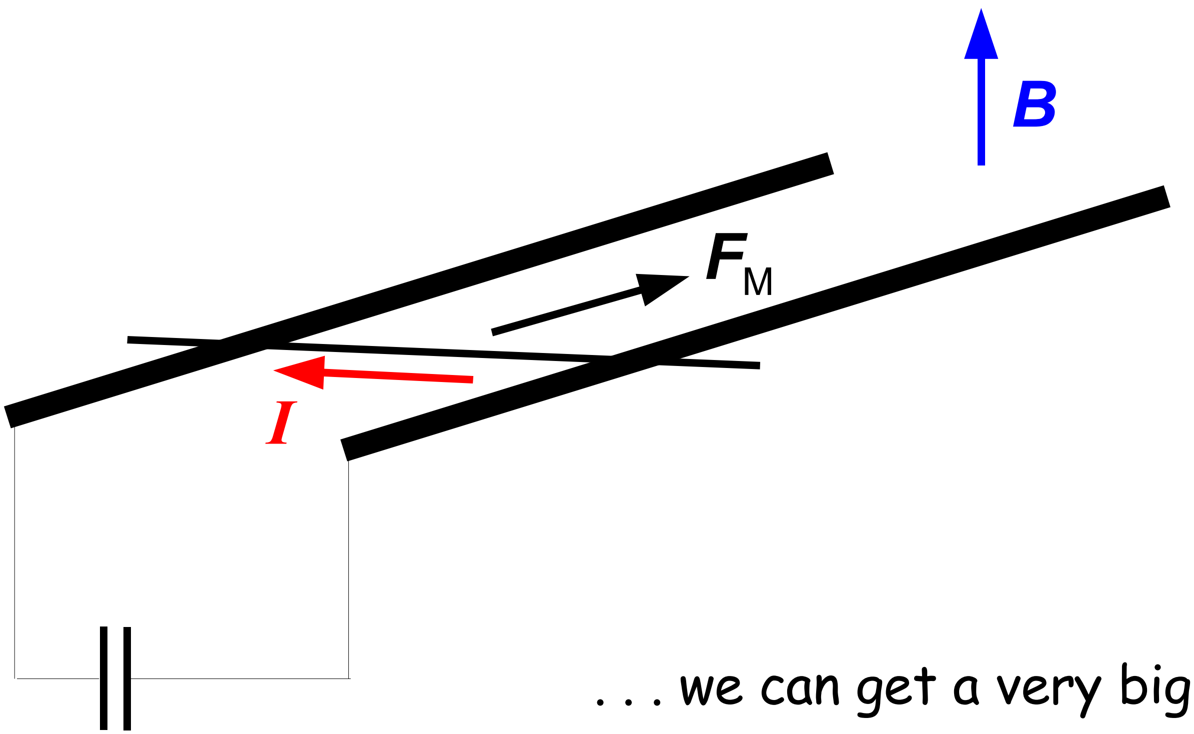
$$nq = JBd/V_{Hall}\,.$$

This can be used to measure both q and n.

Rail Guns

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

If we discharge a capacitor to make a huge current pulse . . .



. . . we can get a very big F_M . . . .

Escape velocities have been achieved, but the projectiles burn up in the air.