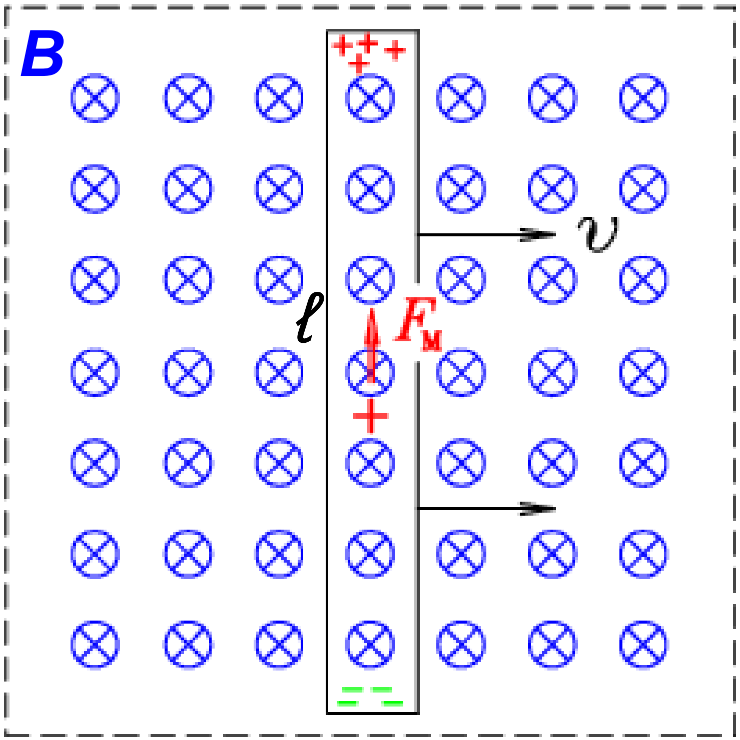
"Derivation" of Faraday's Law

We start with the Lorentz force on the charges in a conducting bar that moves through a uniform magnetic field $\mathbf{B}$ at a speed $\mathbf{v} \perp \mathbf{B}$.

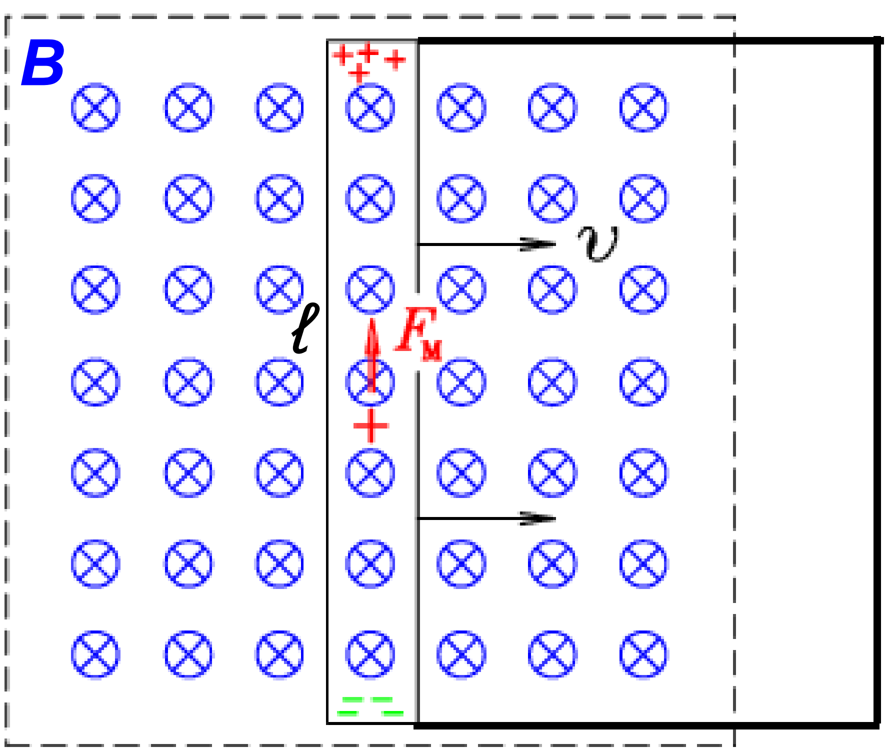
A **+** charge moving with the bar experiences an upward Lorentz force (recall the **Hall effect**) and will move to the top of the bar until enough **+** charges pile up at the top (& **-** charges at the bottom) to create an electric field $\mathbf{E}$ whose force cancels $\mathbf{F}_M$. For a bar of length ℓ, the voltage between the ends is $V = \ell E$ or $V = \ell v B$. But ℓv is the **area** swept out by the bar per unit time, $\ell v = dA/dt$.



Time to define magnetic flux!

Flux through a Loop

We have $V = \ell v B$ where ℓv is the **area** swept out by the bar per unit time, $\ell v = dA/dt$. If we define the **magnetic flux** $\Phi_M = \int \mathbf{B} \cdot d\mathbf{A}$ of the magnetic field passing through the closed loop formed by the bar and the rectangular path shown in black. This loop may be made of physical wires or it may only exist in our imagination; either way, the flux through it is changing at a rate given by $d\Phi_M/dt = \ell v B$. This is the same as the voltage across the bar. Since no other voltages act, it is also the voltage **around the loop**:



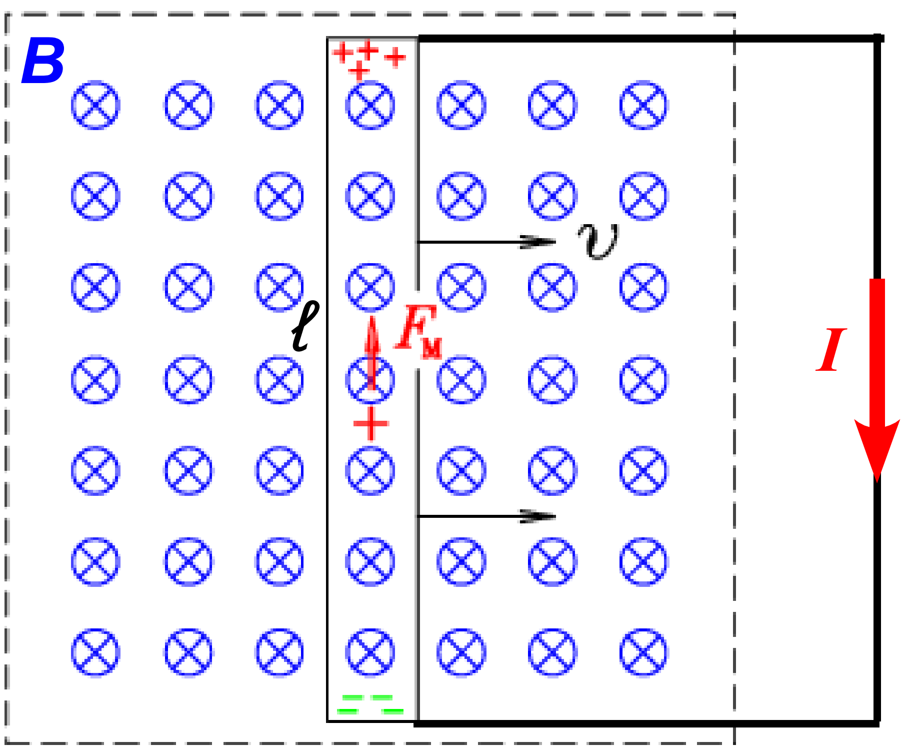
$$\mathcal{E}_{ind} = - d\Phi_M/dt$$

i.e. **Faraday's Law** !

(What does that – sign signify?)

Lenz's Law

We have "derived" **Faraday's Law**, $\mathcal{E}_{ind} = - d\Phi_M/dt$. What does the - sign signify? This has to do with the **direction** of the induced EMF $\mathcal{E}_{ind}$: If we imagine for the moment that the black path actually is a **wire**, then a **current** can flow around the loop due to $\mathcal{E}_{ind}$. (We imagine the wire to have a small **resistance**, to avoid confusing aspects of superconductivity.) The current will flow **clockwise** here, so as to reunite the + & - charges. This current makes its own magnetic field into the page, and thus **adds** to the net flux through the loop in that direction, which **was decreasing** as the loop was pulled out of the field. So <u>the induced EMF "tries" to make a current flow to **counteract the change in flux**</u>. (Lenz's Law)



Magic!

We have "derived" **Faraday's Law**, $\mathcal{E}_{ind} = - d\Phi_M/dt$, and **Lenz's Law** for the particular scenario shown below. This may seem an artificial way of expressing what is obvious from the simple Lorentz force law. What's so amazing about a simple change of terminology? The "magic" of this Law is that it applies (and works!) equally in situations that bear no resemblance to this example. If there is **no physical motion** at all, but only a change in the **strength** of the magnetic field, we still get an induced EMF according to the same equation. This accounts for the enormous impact of "electric power" on the modern world. It also leads to our understanding of the nature of light itself.