Relativistic Shifts of g_μ in Muonic Atoms

Jess H. Brewer

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The discovery of an M9B tune giving substantial transverse muon spin polarization allows convenient μ^-SR experiments in high transverse magnetic field. This new capability has made possible precise measurements of the magnetogyric ratios of negative muons in the ground states of muonic atoms of ^{12}C, ^{16}O, ^{24}Mg, ^{28}Si, ^{32}S, ^{40}Ca, ^{nat}Ti, ^{nat}Zn, ^{nat}Cd and ^{nat}Pb. The precision for ^{12}Cμ^- is ± 23 ppm, of which only 6 ppm is statistical; for ^{nat}Znμ^- the precision is ± 269 ppm and for ^{nat}Pbμ^- it is ± 0.23%. Such experiments may eventually provide a new testing ground for QED in very strong Coulomb fields; today they offer a new way of measuring finite nuclear size effects. I will discuss a little of the history of TRIUMF Experiment 932 and explain how these measurements emerged "accidentally" from an attempt to improve the performance of μ^-SR experiments in general.

Phys. Rev. A Brief Report, in press (2005)

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Bruce A. Fryer	*Chatelech Secondary School*
Donald Arseneau	*TRIUMF (CMMS)*

Facility & Method used:

μSR — otation / elaxation / esonance

m	s	Applied*
u	p	Elementary
o	i	Particle
n	n	Physics

*(to basic research in
Materials Science
and Chemistry)

[and "Fundamental" Physics]

Visit **http://musr.org**

"Themes" in $\mu^{\pm}SR$

μ^+ only (?)

Muonium as light Hydrogen

(Mu = μ^+e^-) (H = p^+e^-)

- ***Mu* vs. *H* atom *Chemistry*:**
  - *gases, liquids & solids*
  - *Best test of reaction rate theories.*
  - *Study "unobservable" H atom rxns.*
  - *Discover new radical species.*
- ***Mu* vs. *H* in *Semiconductors*:**
  - *Until recently, μ^+SR ⟶ **only** data on metastable H states in semiconductors!*

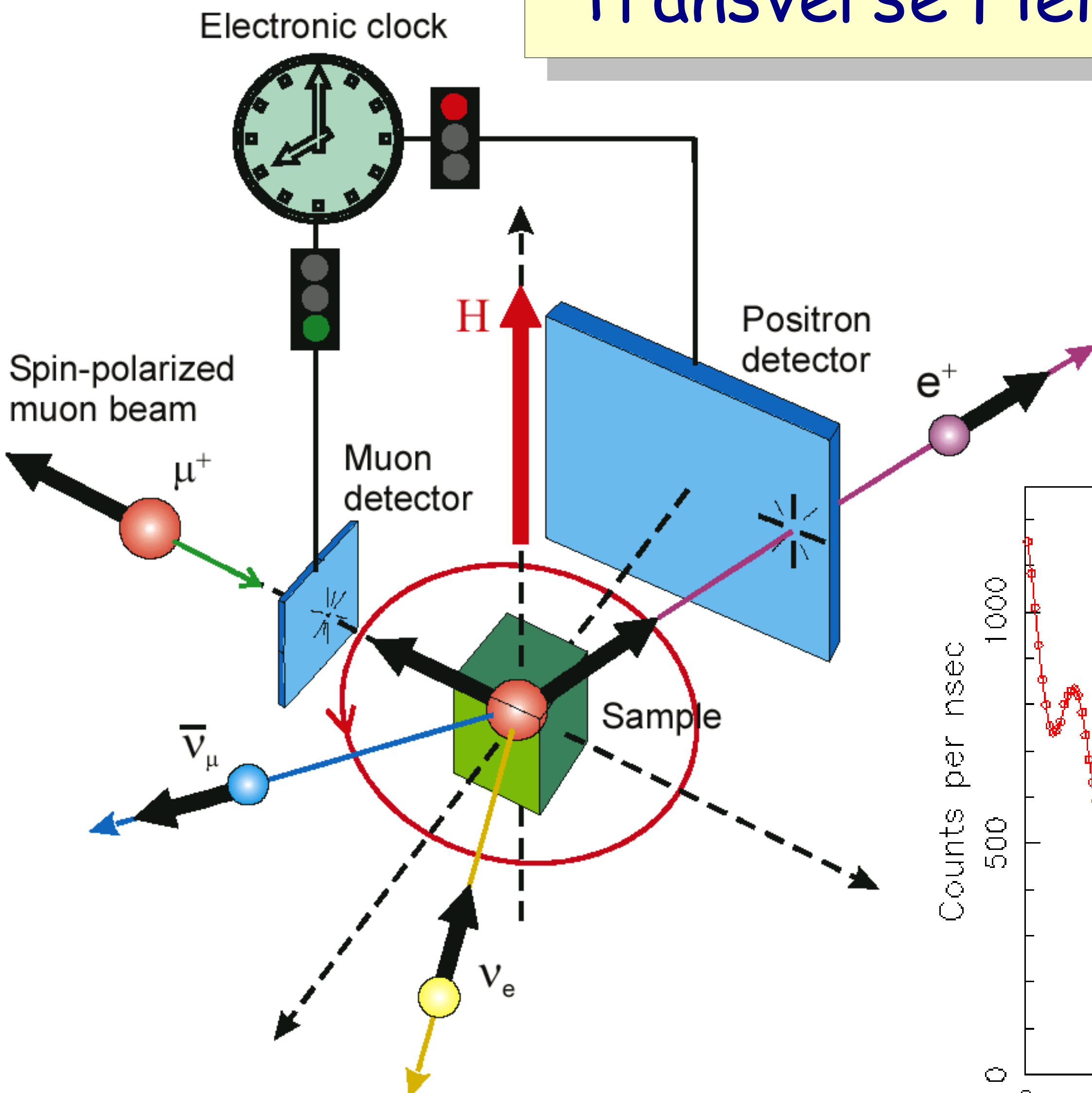
μ^+ or μ^-

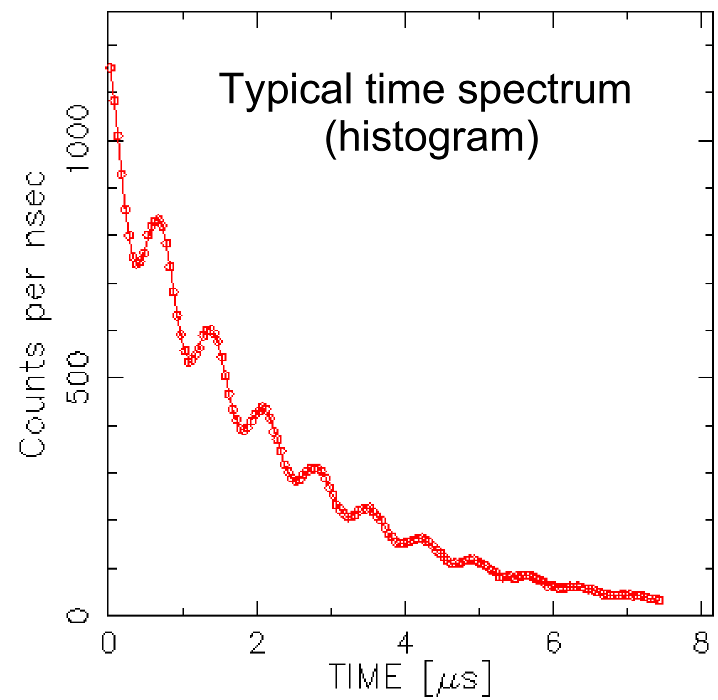
The Muon as a Probe

- **Probing Magnetism:** unequalled sensitivity
  - Local fields: electronic structure; ordering
  - Dynamics: electronic, nuclear spins
- **Probing Superconductivity:** (esp. HT_cSC)
  - Coexistence of SC & Magnetism
  - Magnetic Penetration Depth λ
  - Coherence Length ξ

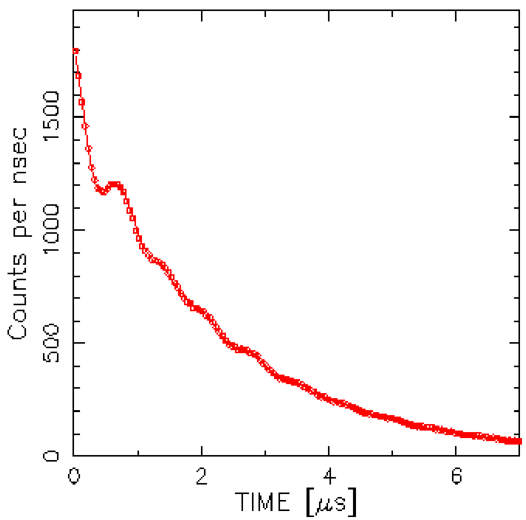
- ***Quantum Diffusion:*** *μ^+ in metals (compare H^+); Mu in nonmetals (compare H).*
- ***Ultra-Heavy Hydrogen:*** *neutral muonic helium ($\alpha^{++}\mu^-e^-$) has $m \approx 4.11\, m_H$*
- ***Muonic Atoms:*** *HF transitions with nucleus;* *relativistic shifts of g_μ*

Transverse Field (TF) $\mu^{+}SR$





μ^+SR *vs.* μ^-SR

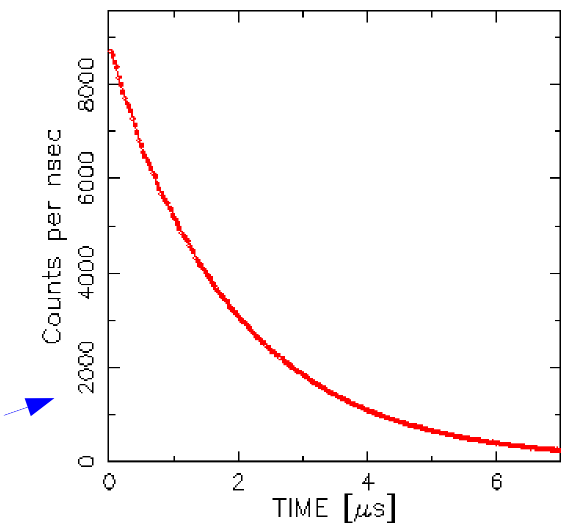


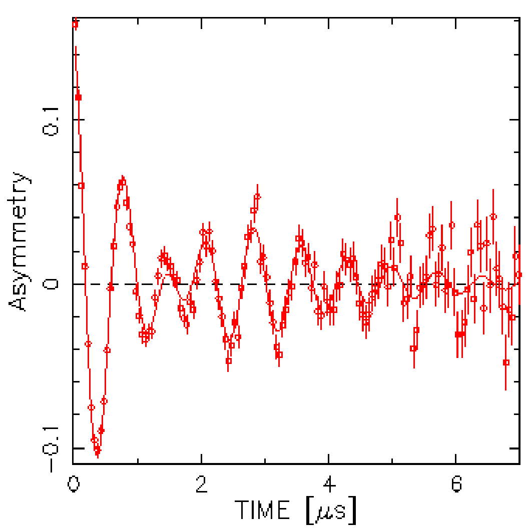
Typical time spectrum (histogram)

Single lifetime

τ_μ = 2.197 μs

Multiple lifetimes (some very short!)

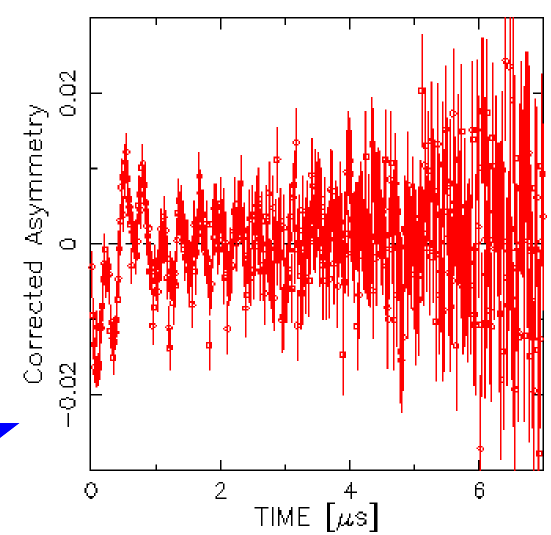




Asymmetry spectrum

Large amplitudes

Small amplitudes



TRIUMF Experiment # ***932***:

Improving $\mu^{-}SR$ Performance

Initial proposal (Dec 2001) to M.S. EEC:
M9B at **LOW** priority(***!***)

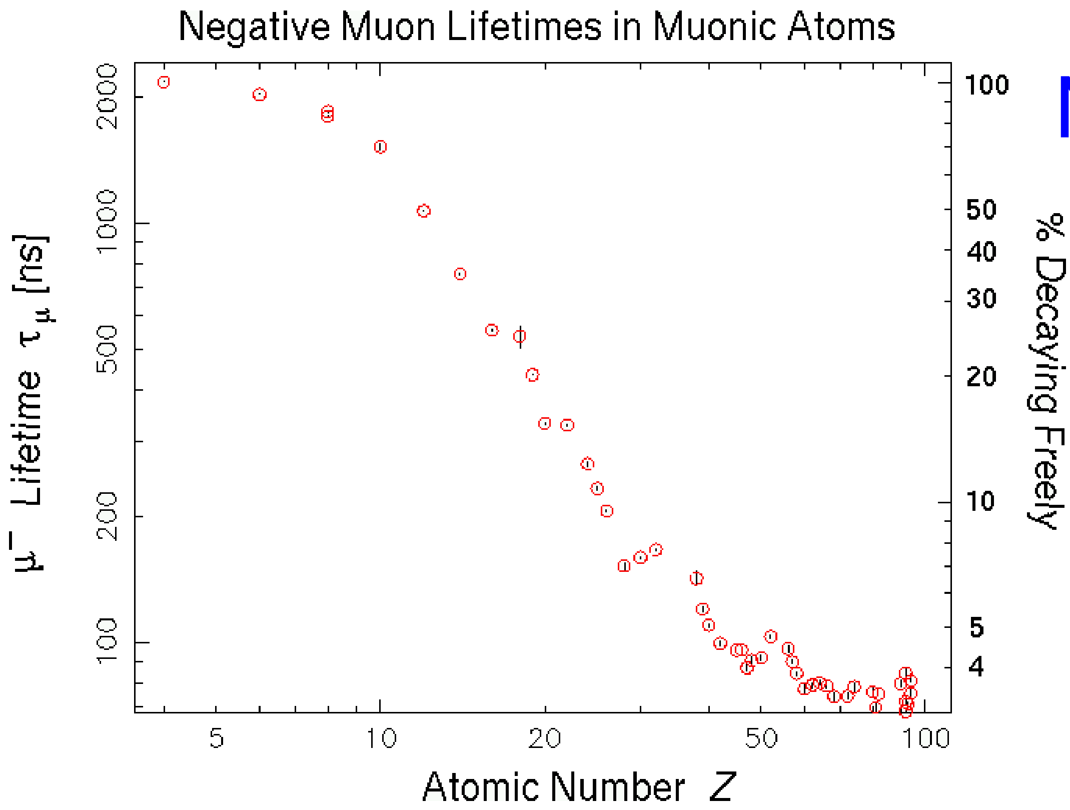
We take ***only*** beam **no one else wants**.

$\mu^- SR$: It is easy to get the impression that only positive muons are employed in μSR.

Although *most* μSR ***is*** $\mu^+ SR$, it is often desirable to use negative muons in the same way, albeit with more difficulty. It is the goal of E**932** to reduce that difficulty.

Drawbacks of $\mu^- SR$ → Proposed Mitigations

- Nuclear Muon Capture: short lifetimes, few decay e^- → • Look for neutron asymmetries in heavier elements ?
- ***L•S*** Depolarization in the atomic cascade → • "Tag" events with specific muonic X-rays ??
- Giant Hyperfine Int. with nonzero-spin nuclei → • Observe characteristic $F^{\pm}$ precession signals ✓



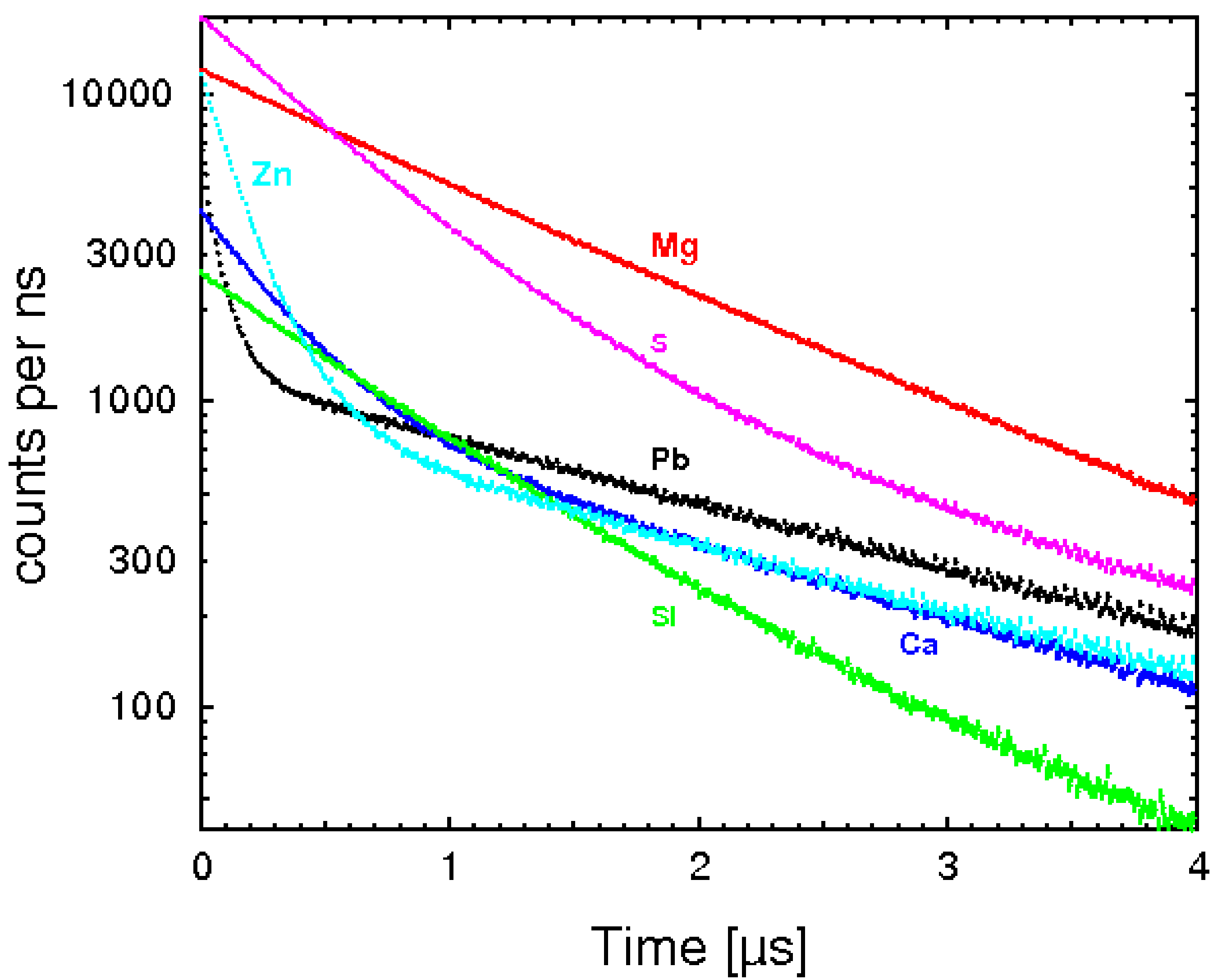
Nuclear μ^- Capture

PROBLEM

$\mu^- p \rightarrow n\, \nu_\mu$ in a nucleus: rate comparable to that of $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$ for $Z \geq 10$.

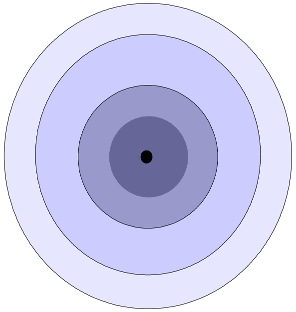
Possible Help: Many times a **fast neutron** is emitted from nuclear muon capture. Very few measurements have been made of the *correlation* of that neutron with the *muon's spin direction*. If cases are found where this **neutron asymmetry** is sizeable, we *may* be able to do neutron-triggered μ^-SR, for which the *event rate* can be *higher* than in μ^+SR.

μ⁻SR Histograms from E932 (M9B, 2004)



Atomic Capture & ***L•S*** Depolarization of μ^-

Large impact parameters are more probable
$\Rightarrow$ initial orbits tend to be fairly ***circular***.

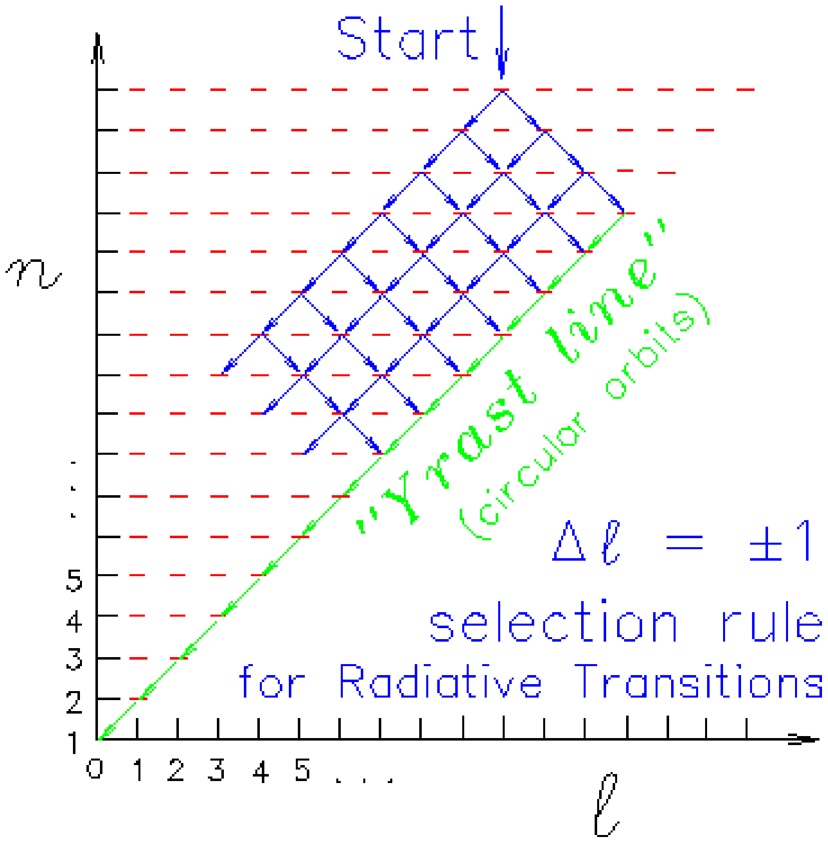


View along μ^- momentum

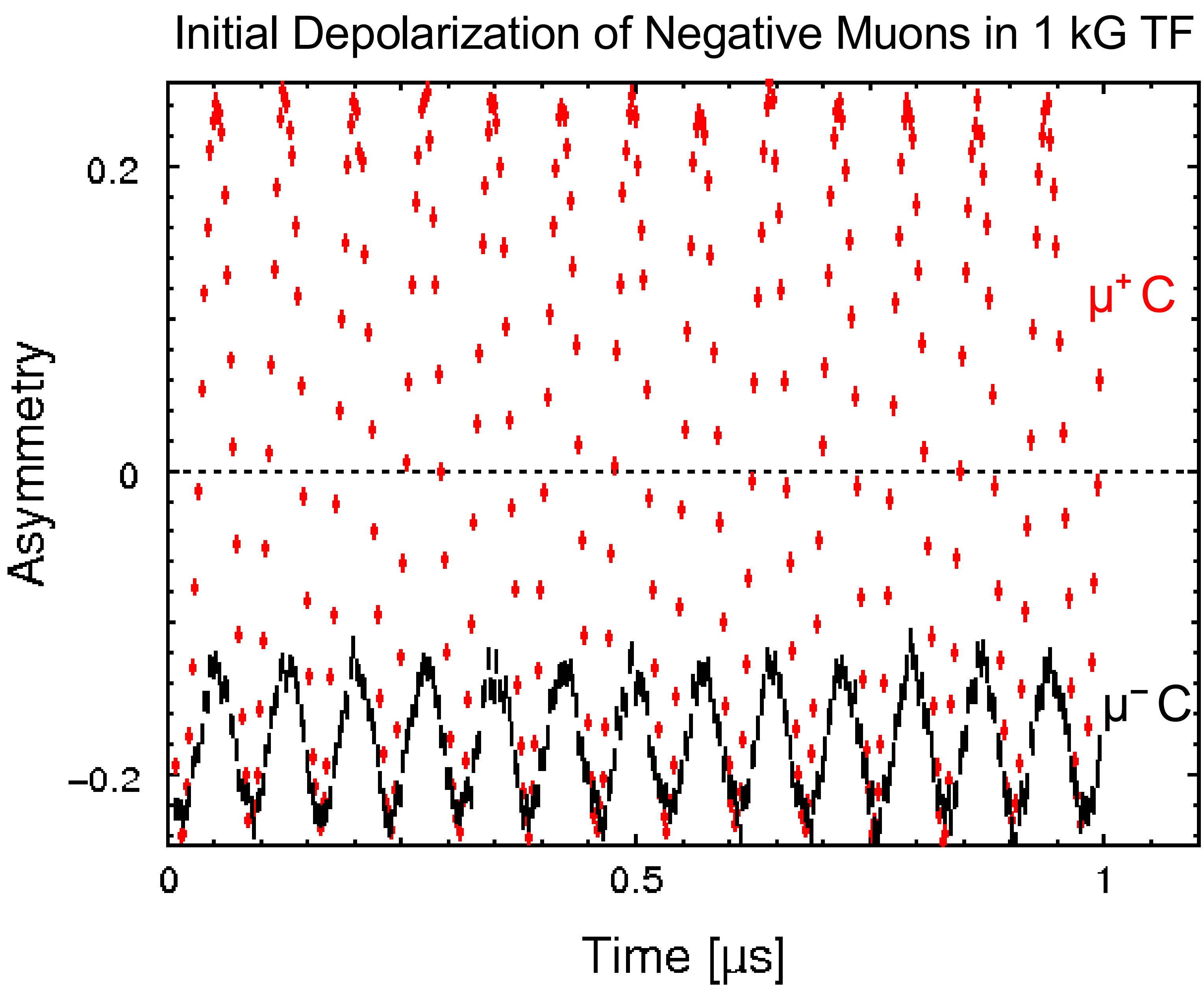
Primitive Atomic Physics:

$$r_n = \frac{a_\circ}{Z}\left(\frac{m_e}{m}\right) n^2$$

$$E_n = -\frac{13.6\text{ eV}}{n^2} Z^2 \left(\frac{m}{m_e}\right)$$



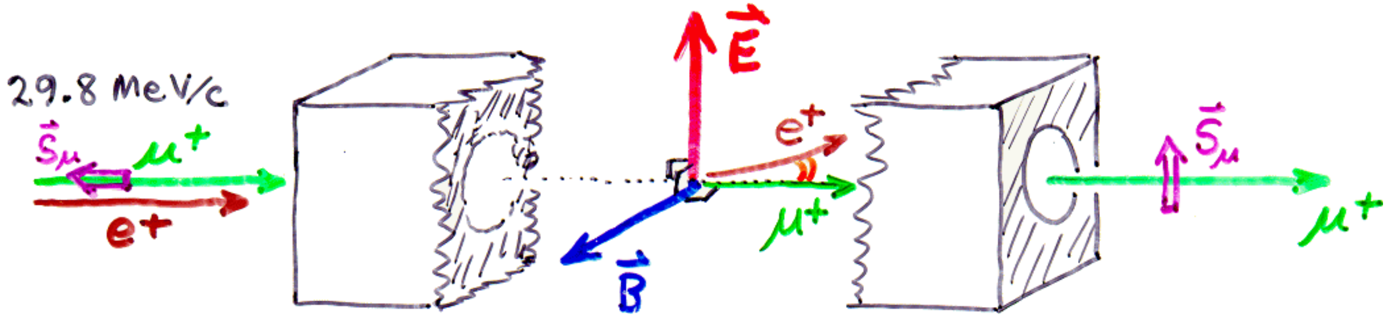
L•S couplings Depolarize μ^- Spin unless ***fast*** Auger!


Initial Depolarization of Negative Muons in 1 kG TF
Asymmetry
0.2
0
−0.2
0
0.5
1
Time [µs]
µ+ C
µ− C

For **positive** "surface muons"

$\mathbf{E}\times\mathbf{B}$ velocity selector

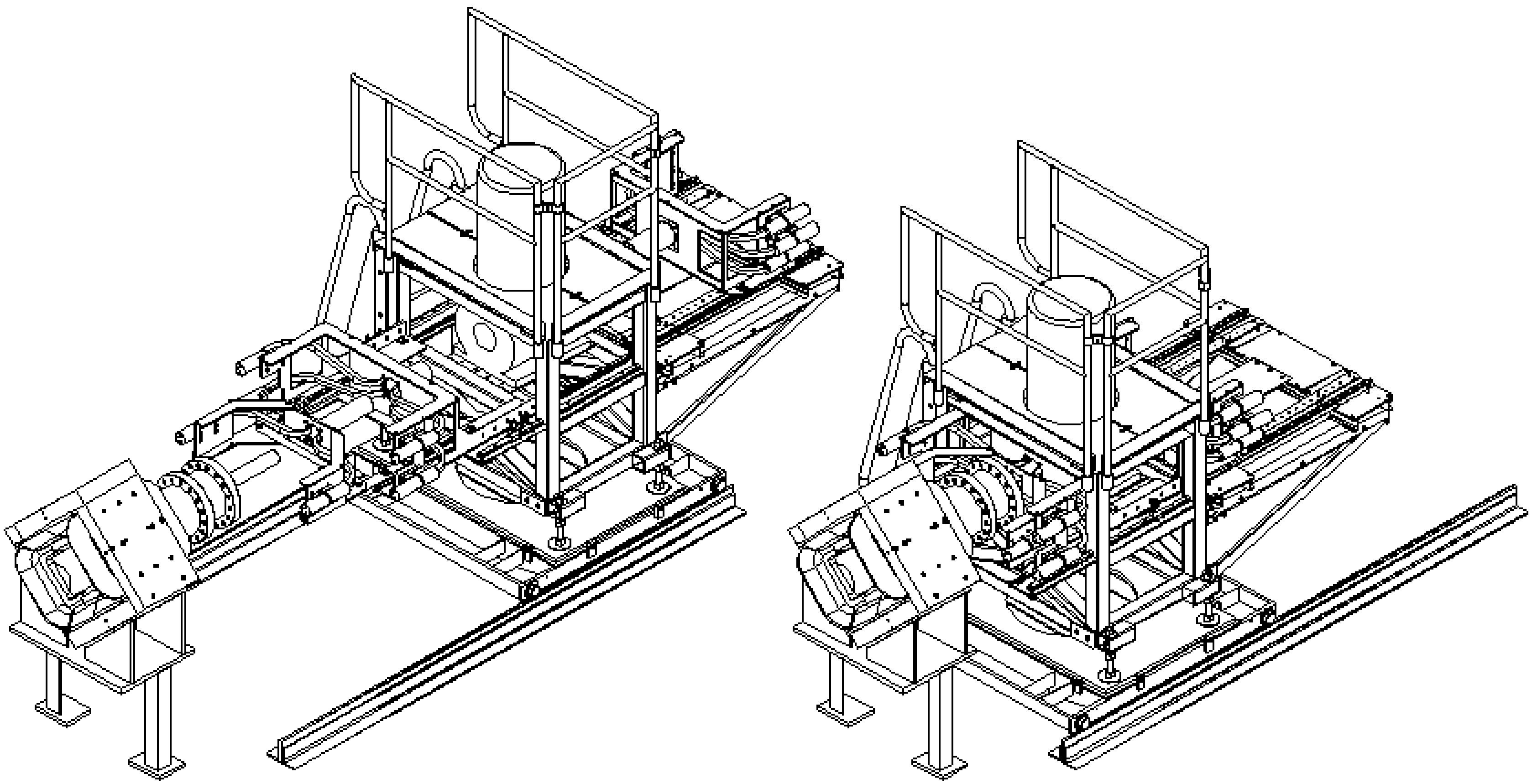
("DC Separator" or Wien filter)



- Removes beam positrons
- Allows TF-μ^+SR in high field (otherwise $B_\perp$ deflects beam)

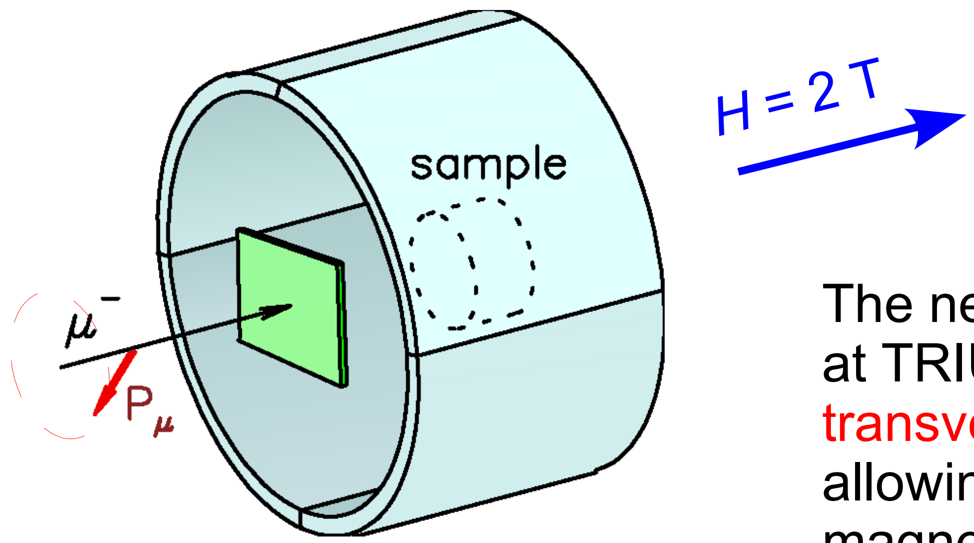
But **not** available for **negative** muons!

Helios



The *Helios* μSR spectrometer of the TRIUMF CMMS facility.

Helios on M9B

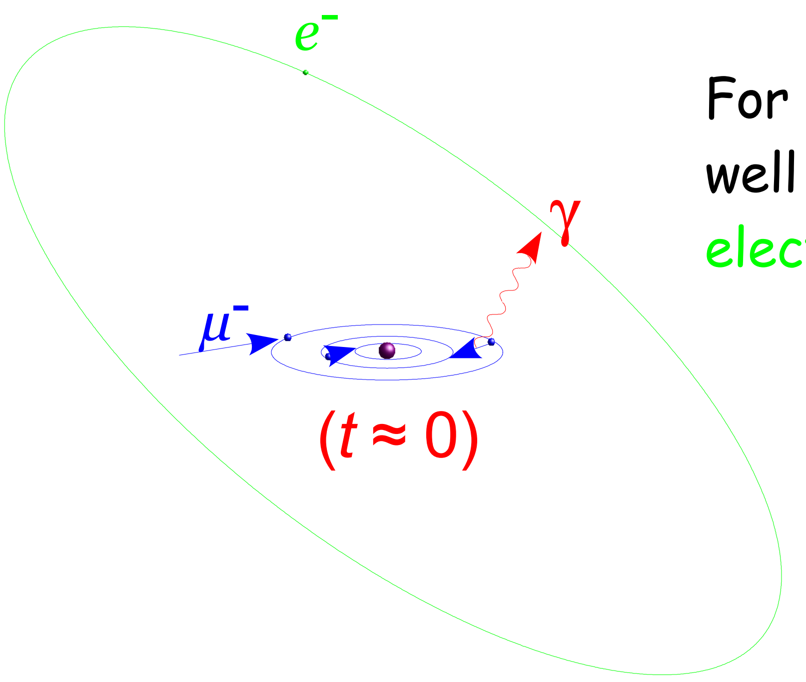


The *Helios* μSR spectrometer of the TRIUMF CMMS facility enables TF-μSR at fields up to 2 T, using 4 *e* detectors in a cylindrical array around the target sample.

The negative muon beam of M9B at TRIUMF has nearly 50% transverse spin polarization, allowing injection into a strong magnetic field parallel to the beam momentum but (partially) transverse to the spins. Strong TF allows high precision measurements of the muon Larmor frequency and thus of g_μ.

Deeply Bound Hydrogenic States

Muonic orbitals are **207** times **smaller** than electronic.



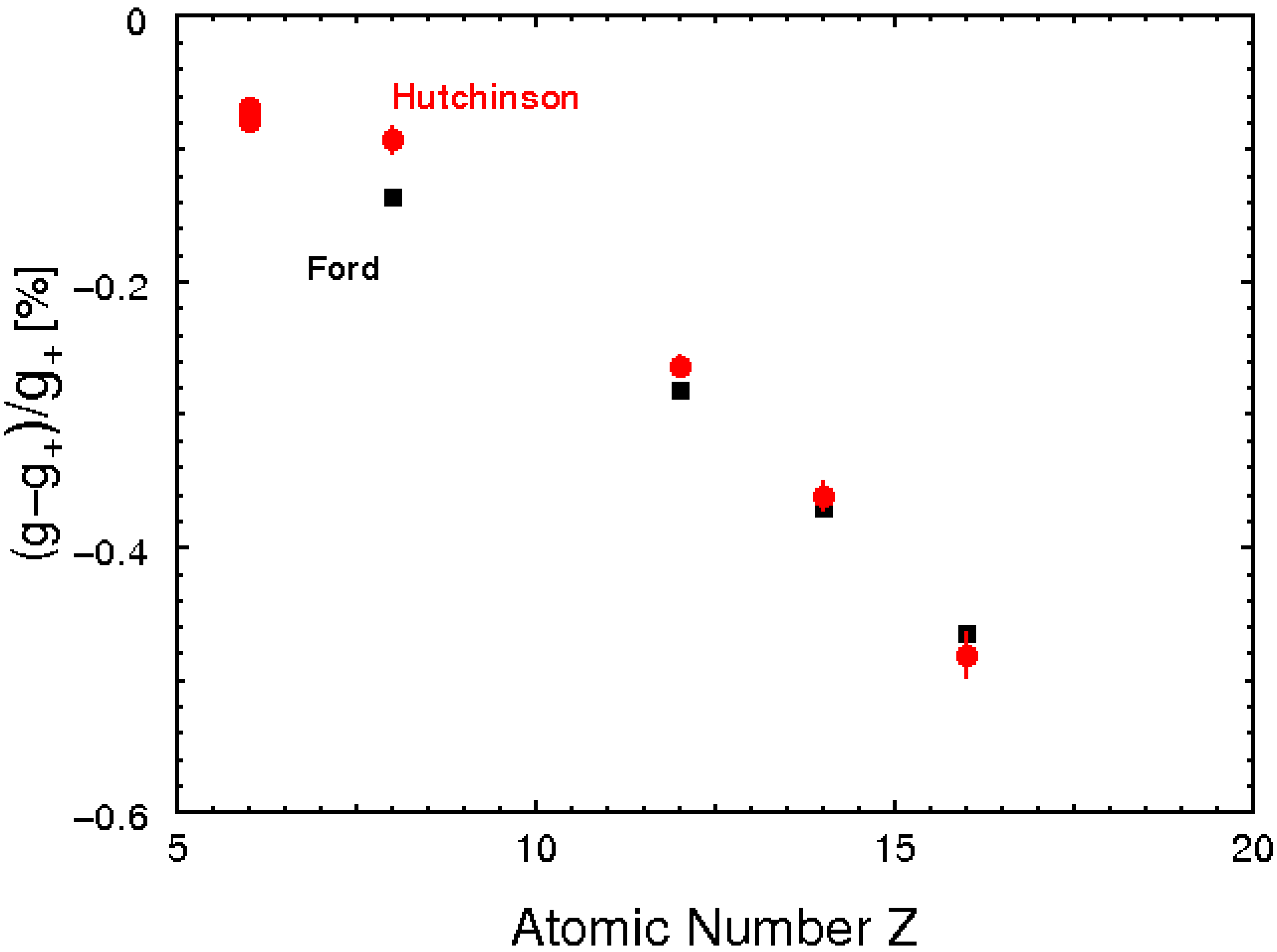
For high Z, the muon is well **inside** even core electronic orbitals.

Relativistic shift of lepton's **magnetogyric ratio** (Breit, 1924): for pointlike nuclei,

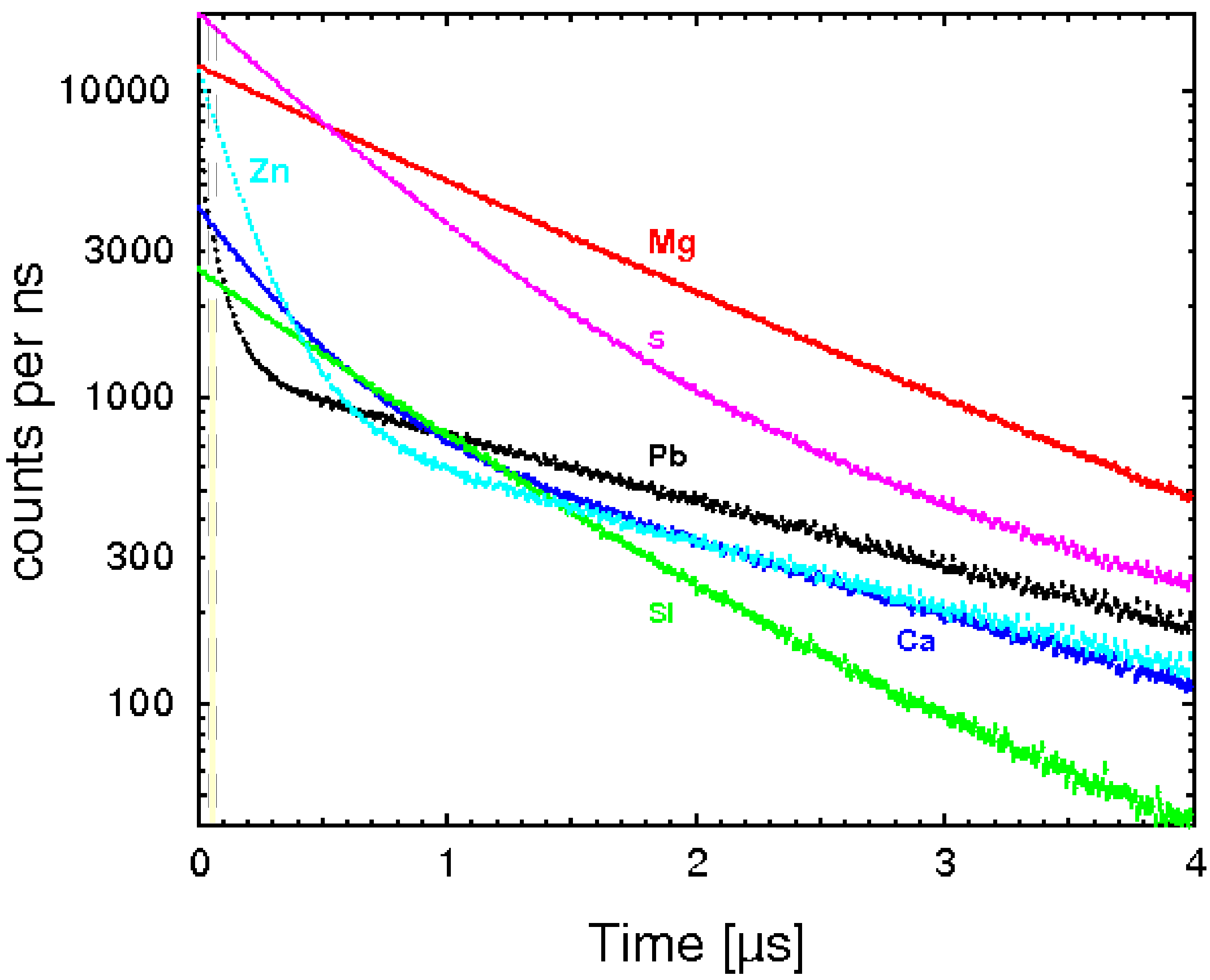
$$\frac{g_{\text{free}} - g}{g_{\text{free}}} = \frac{2}{3}\left(1 - \sqrt{1 - \alpha^2 Z^2}\right) \approx \frac{1}{3}\left(\frac{\bar{v}}{c}\right)^2$$

Theory (Ford et al.) vs. **Expt.** (Hutchinson et al.) in 1961

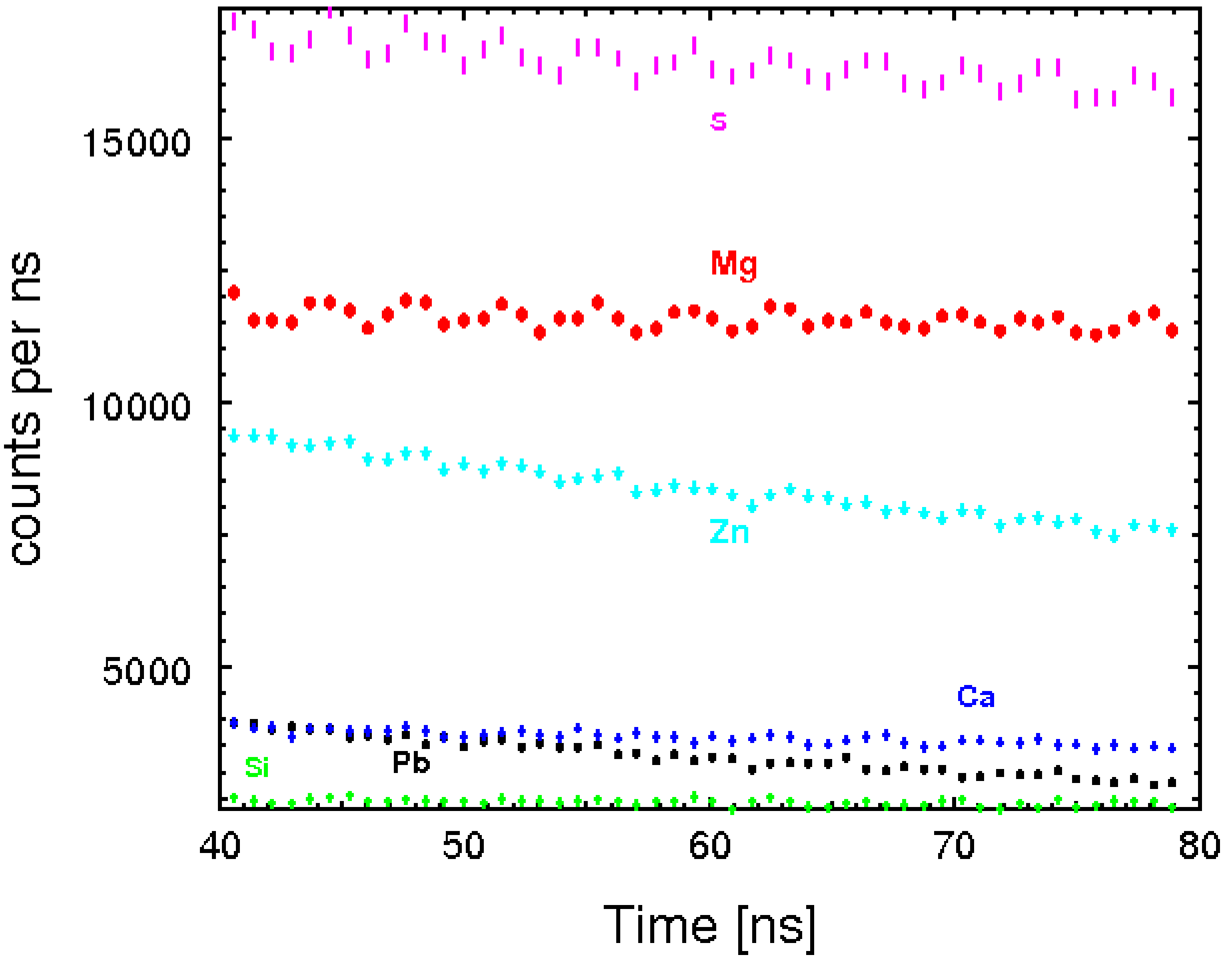
(4 years after the discovery of ***P*** violation in *π*-*μ*-*e* decay)



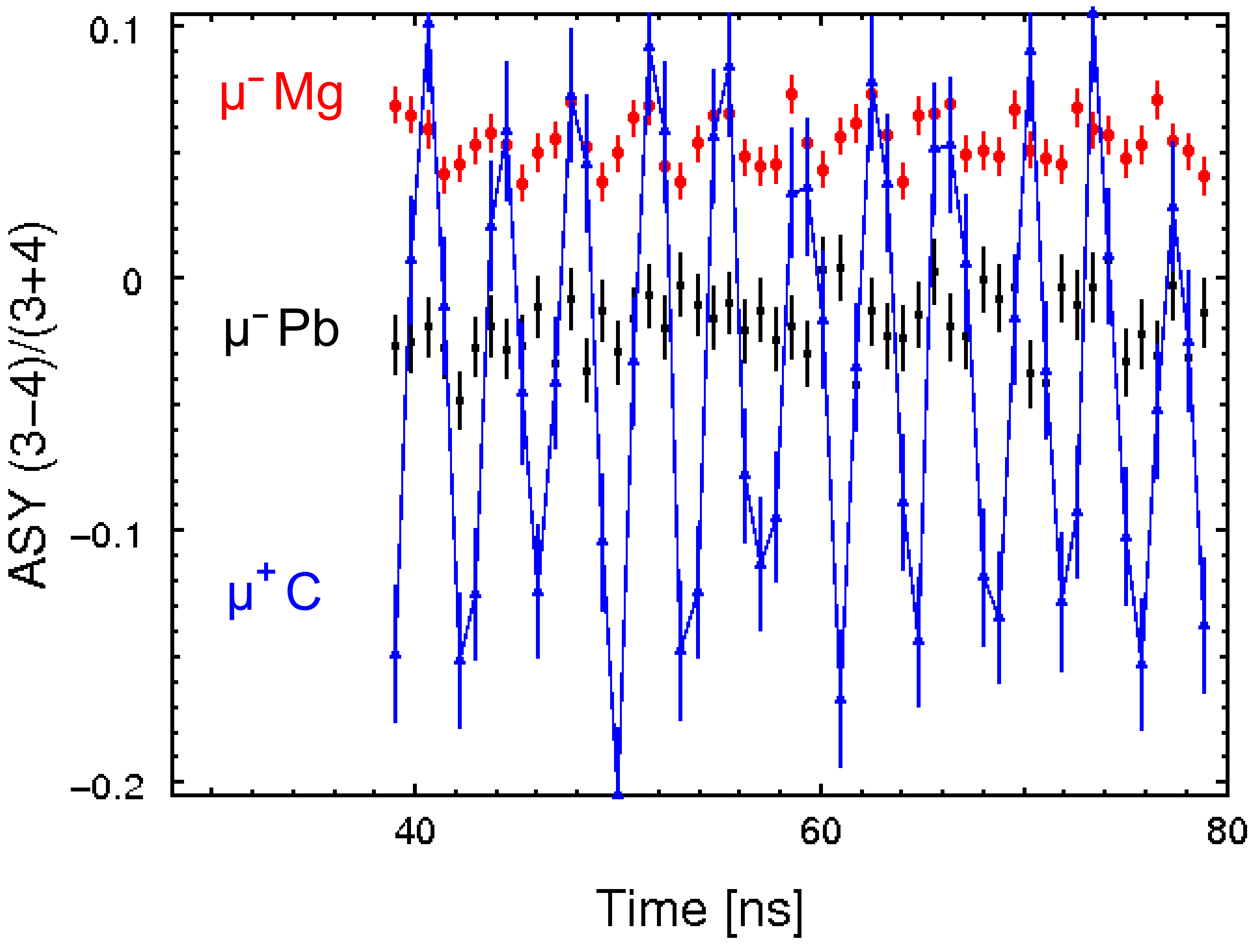
μ⁻SR Histograms from E932 (M9B, 2004)



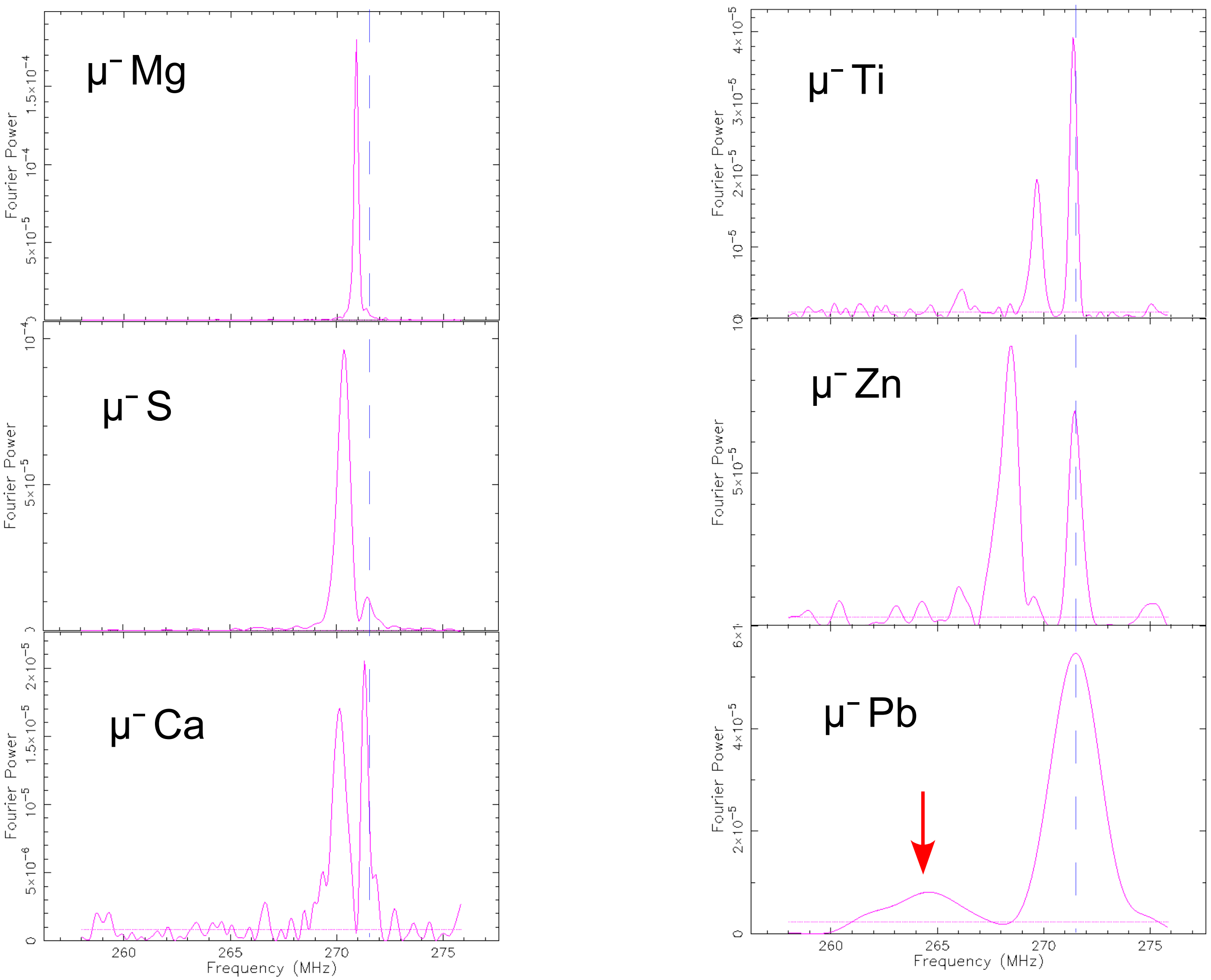
μ⁻ precession signals from E932 (M9B, 2004)



μ⁻ precession signals from E932 (M9B, 2004)



μ⁻ Z frequency spectra from E932 (M9B, 2004)



Raw Data

Sample	Frequency [MHz]
μ^+ in graphite	271.69888 ± 0.00072
μ^+ in Al metal	271.58520 ± 0.00038
μ^- on ^{12}C (graphite)	271.3684 ± 0.0016
μ^- on ^{16}O (H_2O)	271.258 ± 0.010
μ^- on ^{24}Mg (metal)	270.9259 ± 0.0027
μ^- on ^{28}Si	270.6502 ± 0.0069
μ^- on ^{32}S (powder)	270.406 ± 0.008
μ^- on ^{40}Ca (metal)	270.164 ± 0.069
μ^- on Ti (metal)	269.719 ± 0.066
μ^- on Zn (metal)	268.440 ± 0.072
μ^- on Cd (metal)	$265.73\,^{+0.46}_{-0.57}$
μ^- on Pb (metal)	$264.50\,^{+0.59}_{-0.62}$

Only *statistical* uncertainties are shown, to emphasize the potential accuracy of such measurements.

In this experiment, *systematic* uncertainties were dominant for the *light* elements.

Results

Sample	g_μ Shift [%]
μ^+ in graphite	0.0499 ± 0.0023
μ^+ in Al metal	0.0080 ± 0.0004
μ^- on ^{12}C (graphite)	-0.0718 ± 0.0023
μ^- on ^{16}O (H_2O)	-0.1124 ± 0.0042
μ^- on ^{24}Mg (metal)	-0.2348 ± 0.0025
μ^- on ^{28}Si	-0.3363 ± 0.0034
μ^- on ^{32}S (powder)	-0.4262 ± 0.0036
μ^- on ^{40}Ca (metal)	-0.5155 ± 0.025
μ^- on Ti (metal)	-0.679 ± 0.024
μ^- on Zn (metal)	-1.150 ± 0.026
μ^- on Cd (metal)	$-2.15^{+0.17}_{-0.21}$
μ^- on Pb (metal)	$-2.60^{+0.22}_{-0.23}$

Fractional shifts (in %) of the negative muon's *g* factor due to ***relativistic*** effects in the deeply bound ground state of the muonic atom.

(In Pb, most of the muon's orbital lies *inside* the nucleus!)

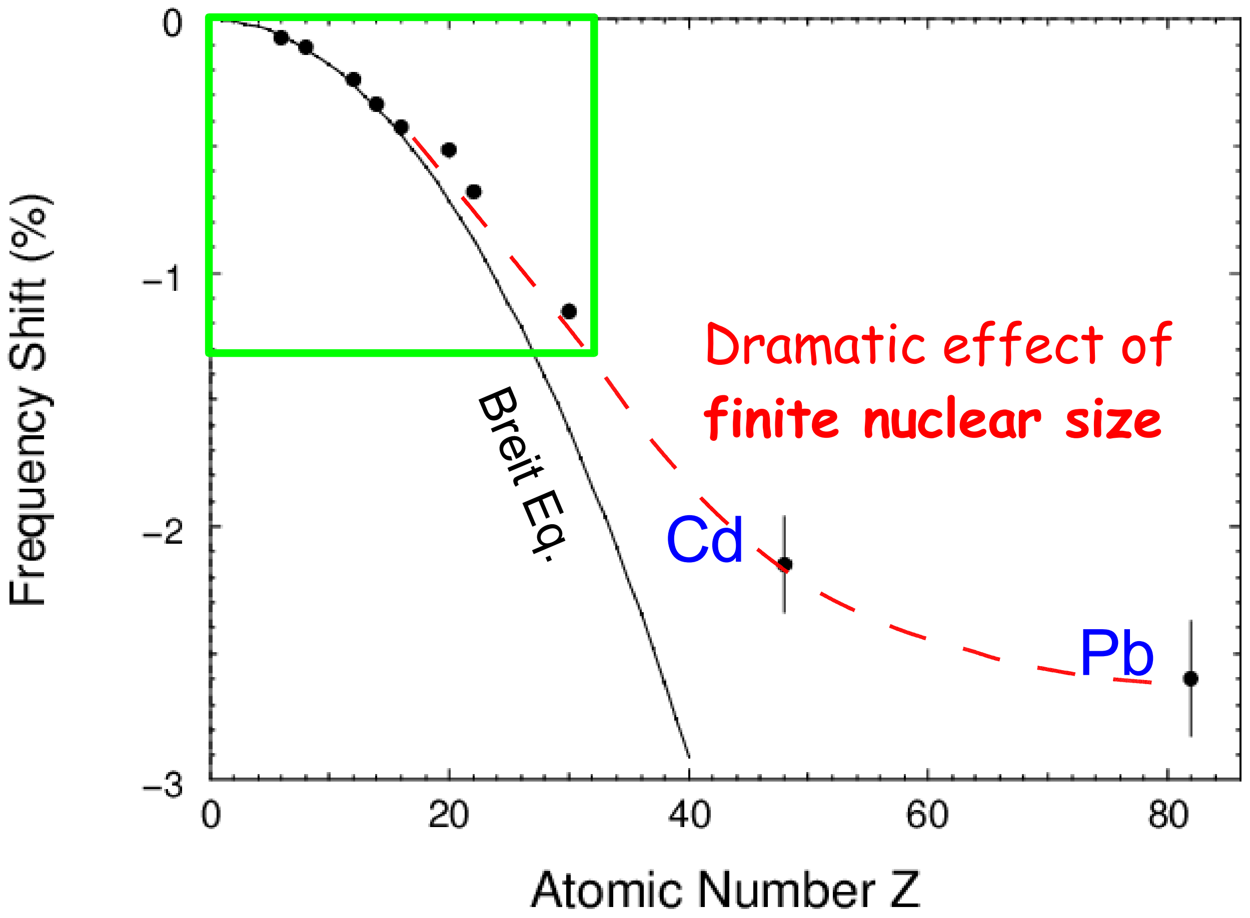
So what does it **mean**?

For pointlike nuclei (Breit, 1928):

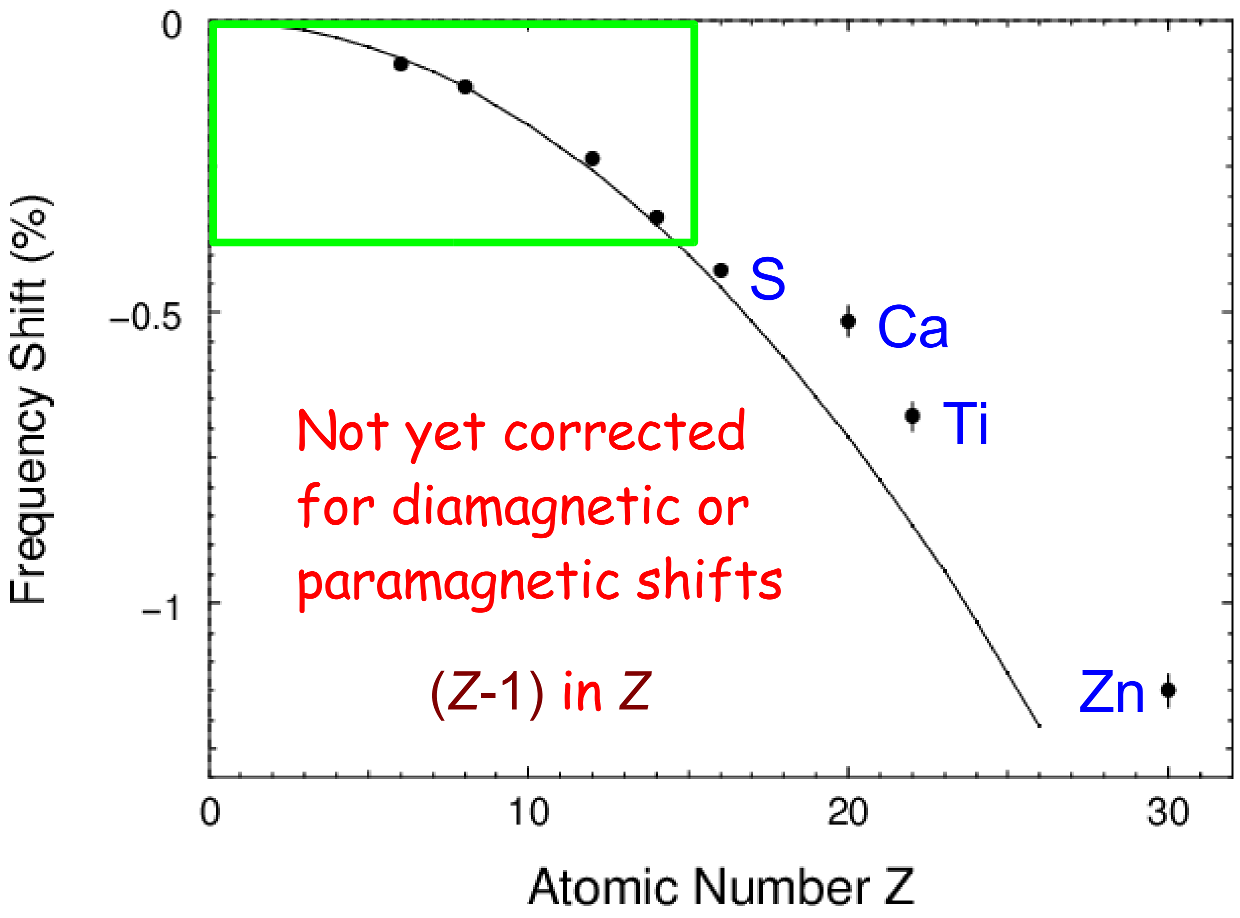
$$\frac{g_{\rm free} - g}{g_{\rm free}} = \frac{2}{3}\left(1 - \sqrt{1 - \alpha^2 Z^2}\right) \approx \frac{1}{3}\left(\frac{\bar{v}}{c}\right)^2$$

Improved by Margeneau (1940) and later by Ford *et al.* (1962) in response to first μ^-SR measurements by Hutchinson *et al.* (1961) in light elements. First high-*Z* measurements by Yamazaki *et al.* (1974) challenged by Mamedov *et al.* (2003). Meanwhile electronic spectroscopy of high *Z* hydrogenlike ions has become possible [*e.g.* Häffner *et al.* (2000)].

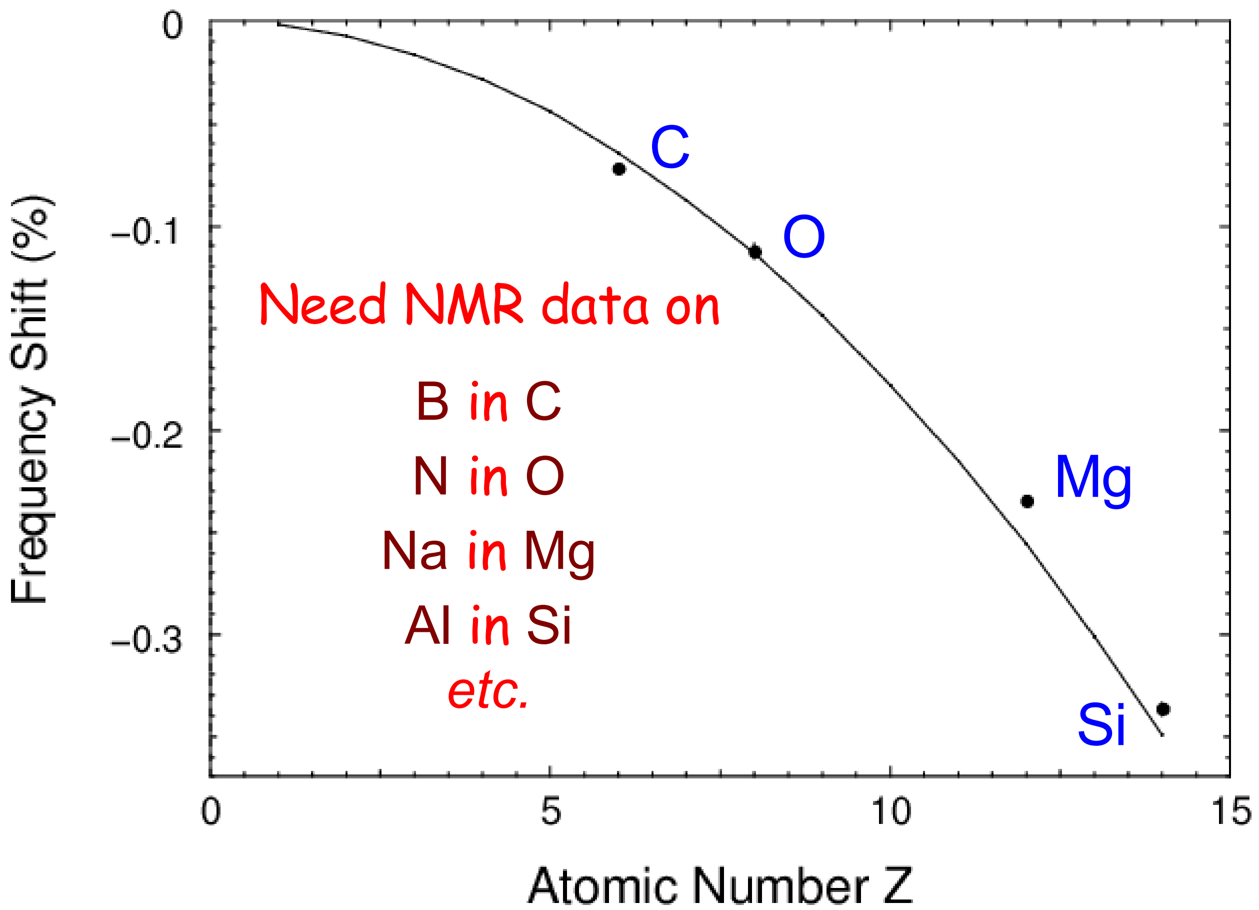
Relativistic Shift of μ^- Frequency



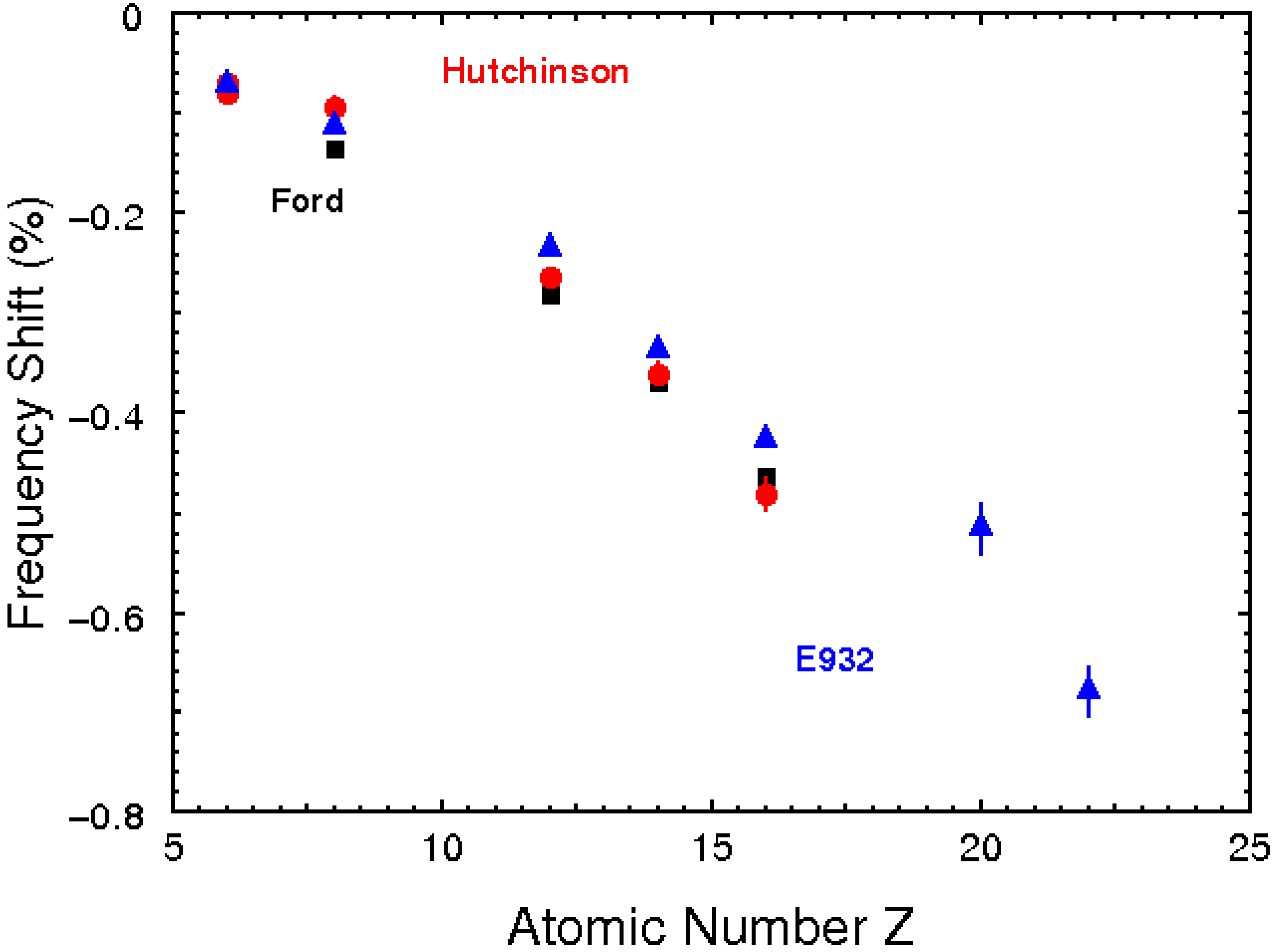
Relativistic Shift of μ^- Frequency



Relativistic Shift of μ^- Frequency



E932 vs. Ford & Hutchinson



Phil Anderson:

(at a High T_c Superconductivity conference)

"Experimentalists should not attempt to interpret their own data."

[paraphrased]

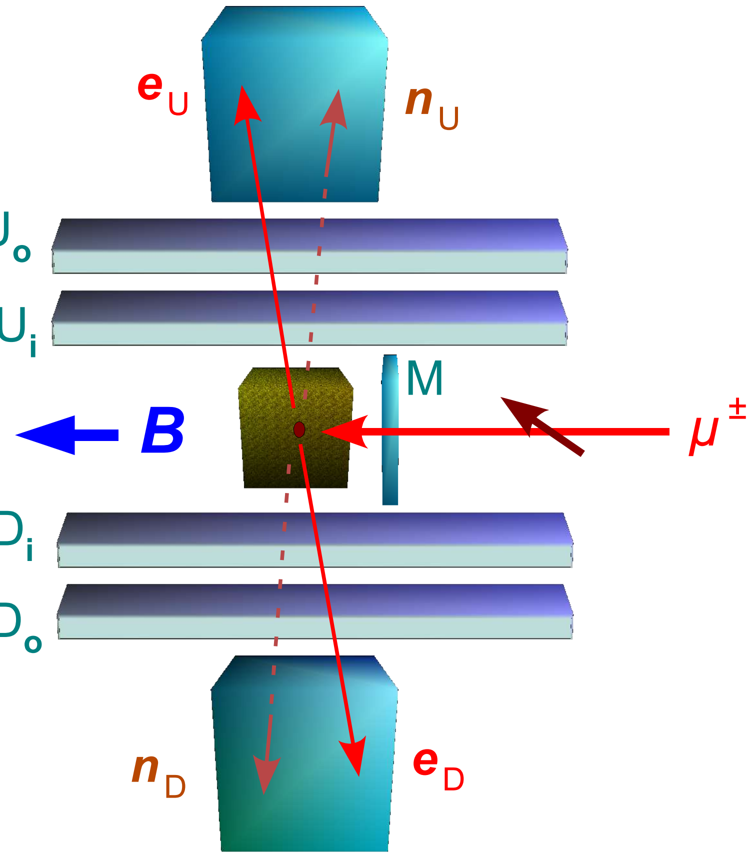
Darth Vader:

"Leave that to me."

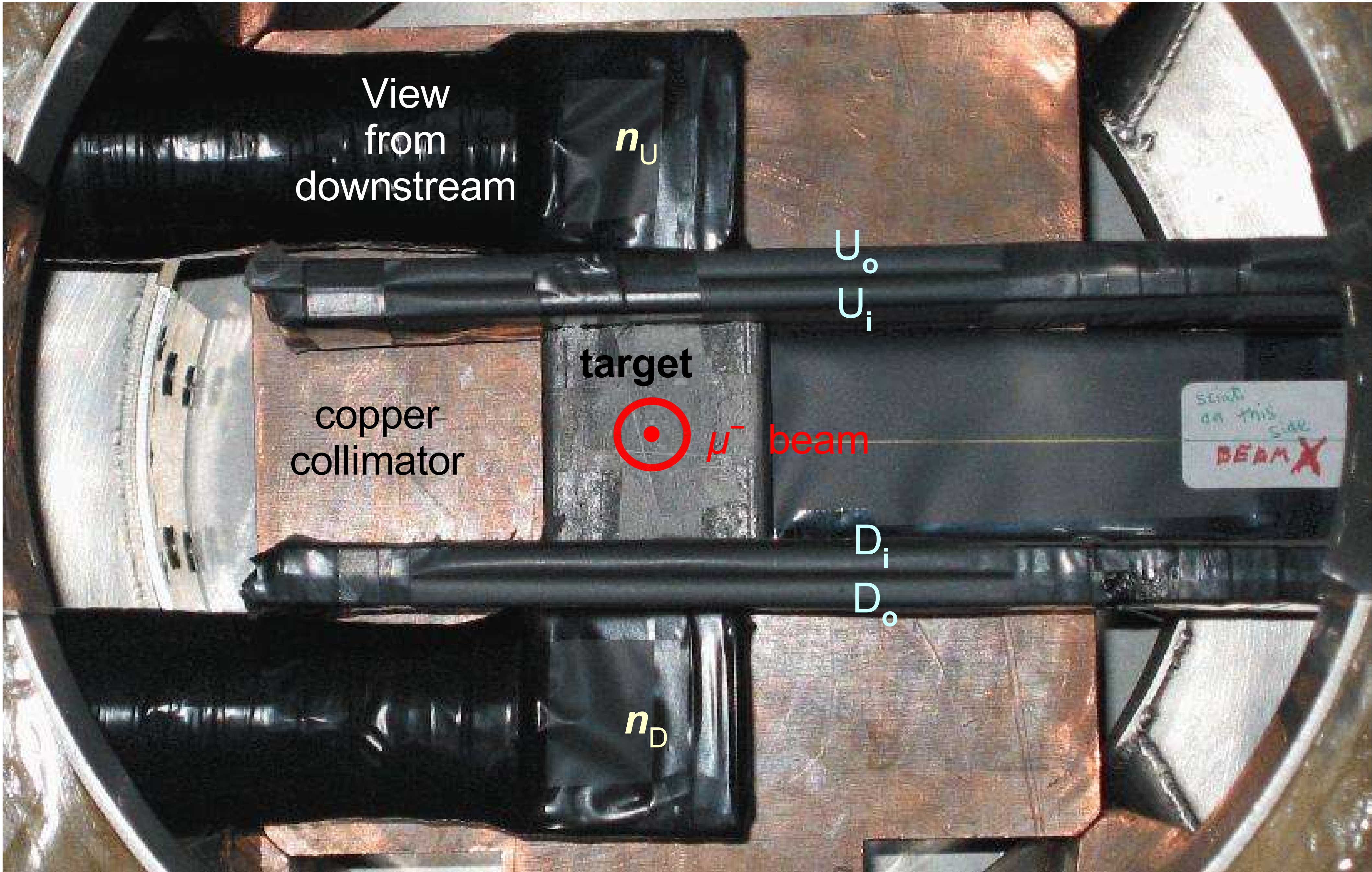
Finis

(for now)

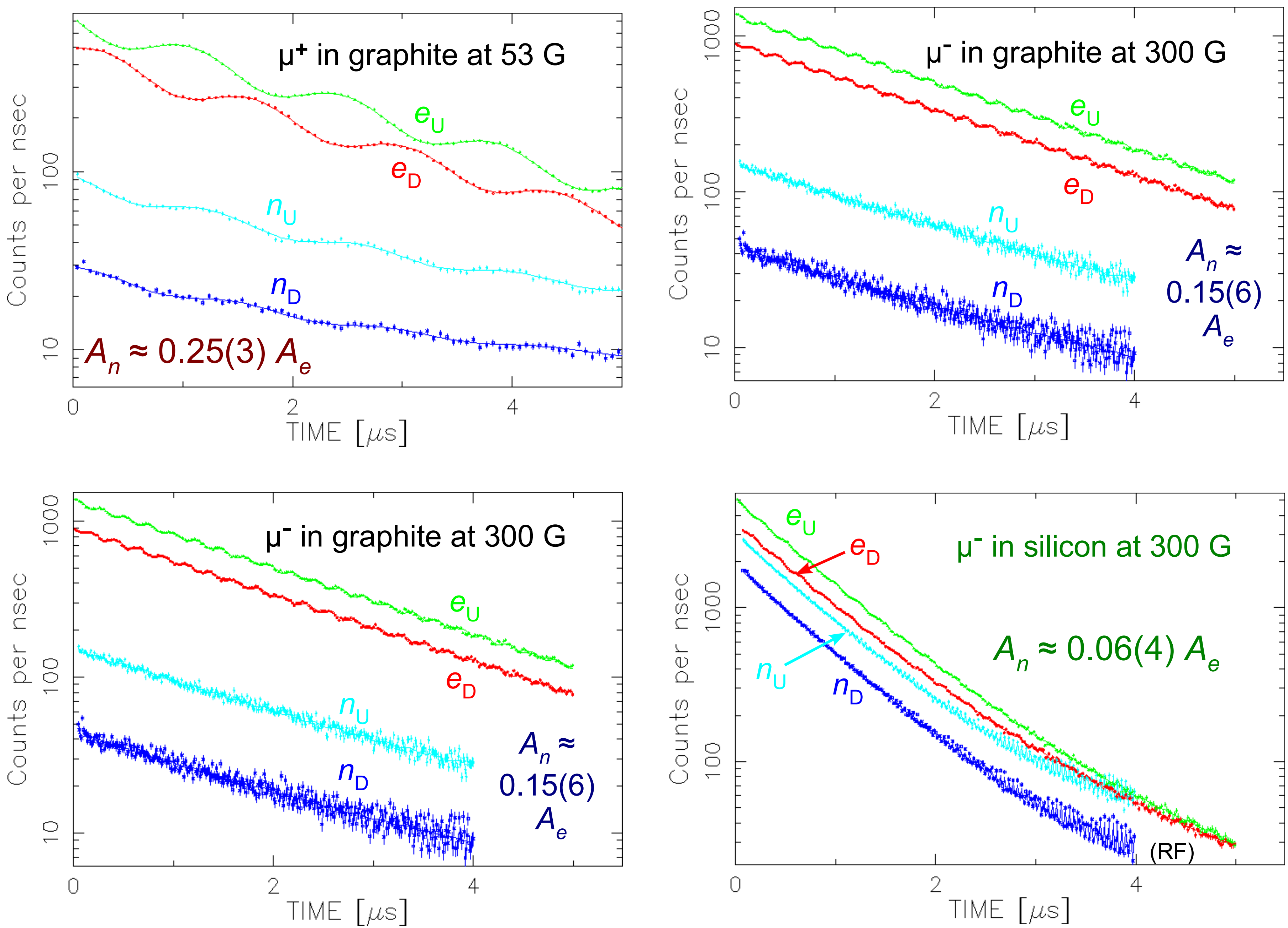
E932 on M9B, July 2005: 1st look at **Neutrons**



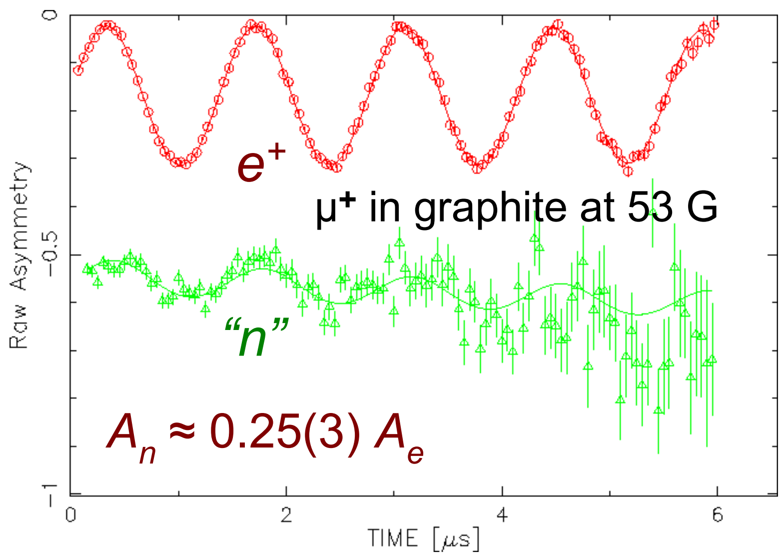
E932 on M9B, July 2005: Neutron counters in Omni'

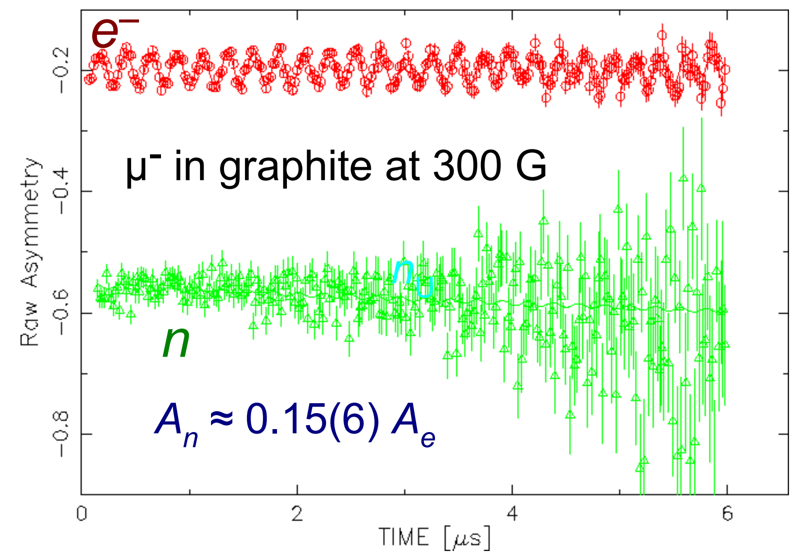


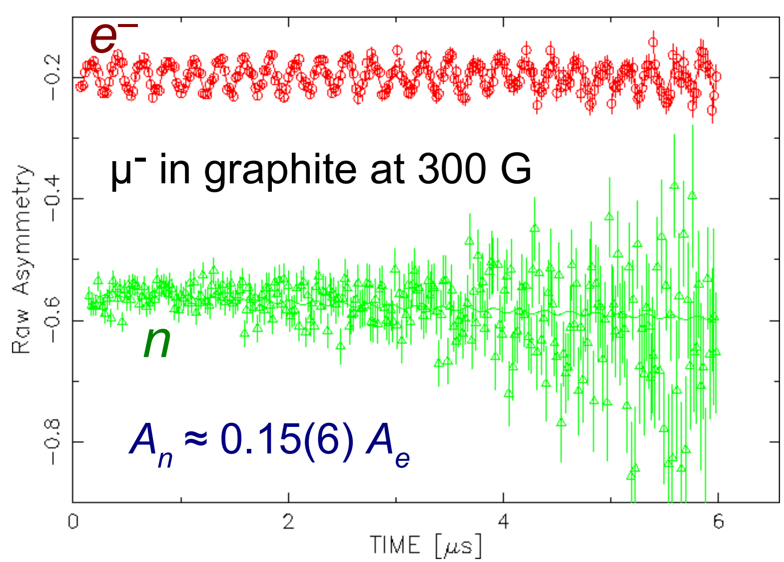
M9B data 2-4 July 2005: Raw Histograms

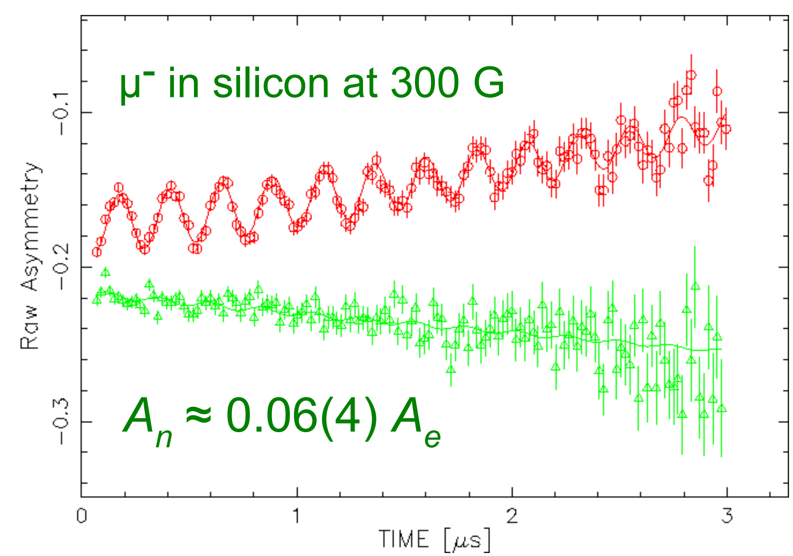


M9B data 2-4 July 2005: (U-D)/(U+D) Asymmetries









Negative muon spin precession measurement of the hyperfine states of muonic sodium

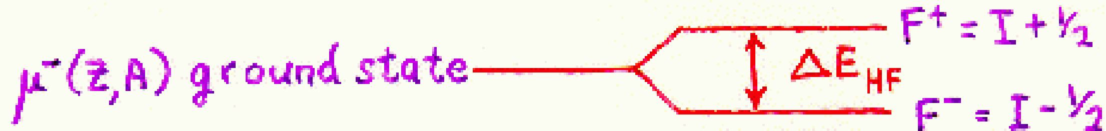
Jess H. Brewer, Khashayar Ghandi,
Aaron M. Froese & Bruce A. Fryer

CIAR, UBC, TRIUMF & Chatelech Secondary School

Both hyperfine states of muonic ^{23}Na and the rate R_{HF} of conversion between them have been observed directly in a high field negative muon spin precession experiment using a backward muon beam with transverse spin polarization. The result in metallic sodium, $R = 13.7 \pm 2.2\ \mu s^{-1}$, is consistent with Winston's prediction in 1963 based on Auger emission of core electrons, and with the measurements of Gorringe *et al*. in Na metal, but not with their smaller result in NaF. In NaOH we find $R = 23.5 \pm 8\ \mu s^{-1}$, leaving medium-dependent effects ambiguous.

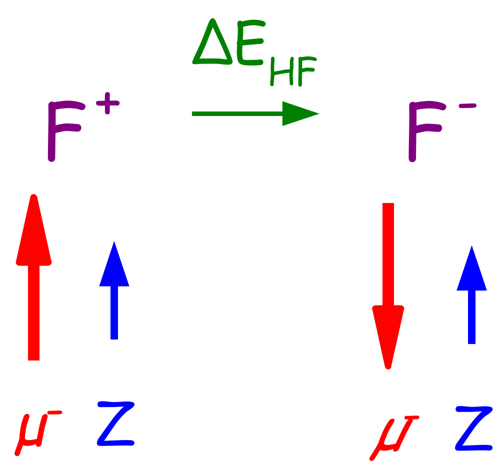
Phys. Rev. C Brief Reports **71**, 058501 (2005).

HFS OF SELECTED "BARE" MUONIC ATOMS:



^{A}ELEMENT$_{Z}$	ΔE_{HF} (eV)	effective field B_{HF} (G)	Lowest e^- SHELL ejected [B.E. (eV)]	$F^+ \to F^-$ RATE R (μs^{-1}) THEOR	EXPT	REF
$^{1}H_{1}$	0.1817	3.24×10^{9}	NONE	1.3×10^{4} (ρ/ρ_0)		Ponomarev '78; Smilga & Fil'chenko '83
$^{3}He_{2}$	1.373	2.45×10^{10}	NONE	nil		
$^{11}B_{5}$	18	3.2×10^{11}	L_1 [9.3]	0.25	0.33(5)	Winston '63; Favart et al '70
$^{13}C_{6}$	11	2.0×10^{11}	L_2 [8.3]		0.016(12)	Boom '83
$^{14}N_{7}$	7.5	1.3×10^{11}	NONE	nil	0.092(33)	"
$^{19}F_{9}$	126	2.2×10^{12}	L_1 [30]	5.8	6.1(7) [neutrons]	Winston '63
$^{27}Al_{13}$	263	4.7×10^{12}	L_1 [89]	41	41(9)	" & Brewer '83
$^{51}V_{23}$	1220	2.2×10^{13}	L_1 [565]	700		Winston '63
$^{93}Nb_{41}$	~5000	~10^{14}				
$^{209}Bi_{83}$	4660	8.3×10^{13}	MEASURED VIA SPLITTING of X-RAY ENERGY			

Characteristic precession frequencies of $F^{\pm}$ hyperfine states in selected low-Z muonic atoms



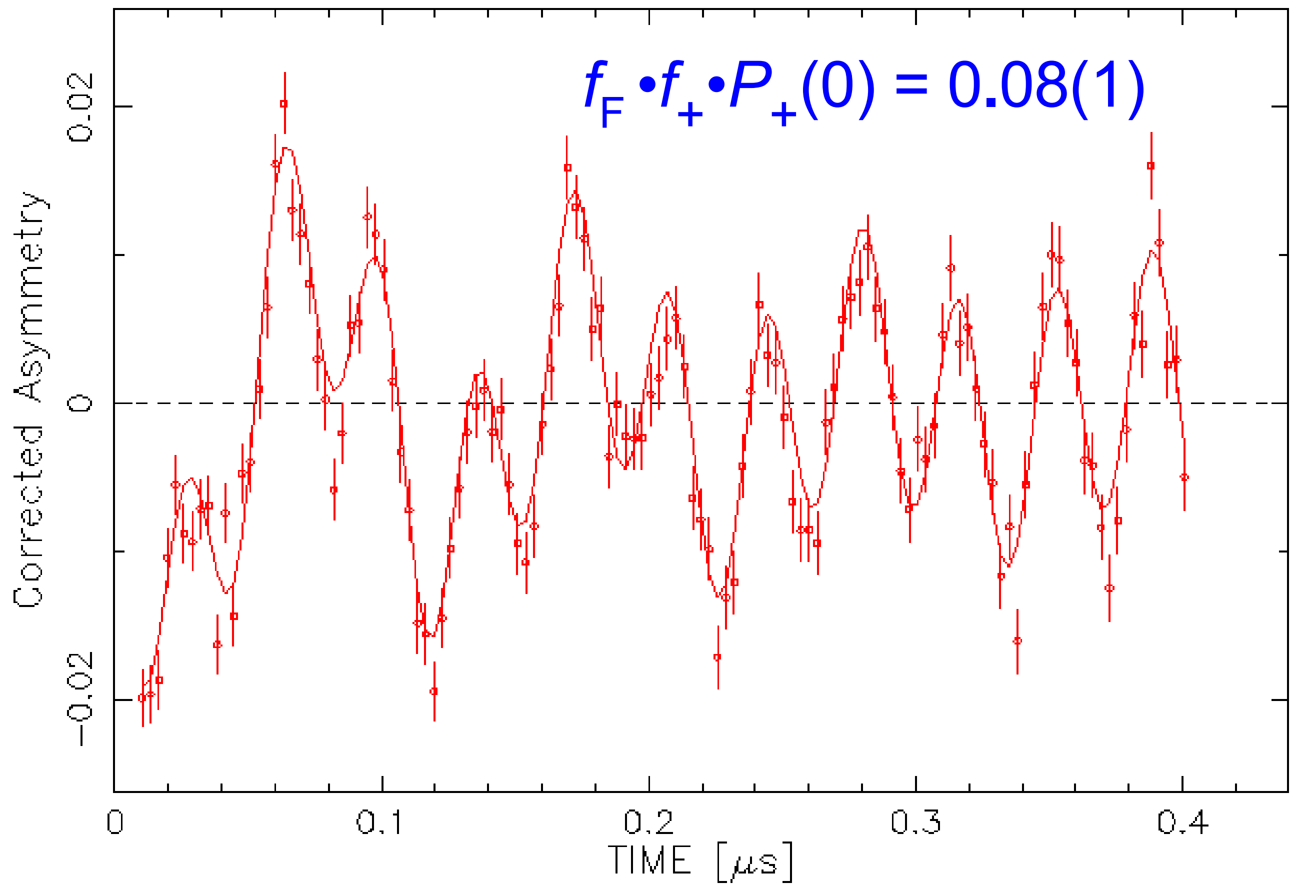
Isotope $^{A}El_{Z}$	Nucl. Spin	Natural Abundance	Moment μ_N/μ_μ	Frequency Ratios F^+/μ^-	F^-/μ^-
$^{1}H_{1}$	1/2	≈ 1	−0.314109	0.342946	0
$^{2}H_{1}$	1	≈ 0	−0.096436	0.301188	−0.397624
$^{6}Li_{3}$	1	0.07	−0.092454	0.302515	−0.394969
$^{7}Li_{3}$	3/2	0.93	−0.366253	0.158437	−0.402606
$^{9}Be_{4}$	3/2	≈ 1	0.132447	0.283112	−0.194814
$^{10}B_{5}$	3	0.19	−0.202528	0.113925	−0.181434
$^{11}B_{5}$	3/2	0.81	−0.302380	0.174405	−0.375992
$^{13}C_{6}$	1/2	0.01	−0.079000	0.460500	0
$^{14}N_{7}$	1	≈ 1	−0.045394	0.318202	−0.363596
$^{19}F_{9}$	1/2	≈ 1	−0.295666	0.352167	0
$^{23}Na_{11}$	3/2	≈ 1	−0.249406	0.187648	−0.353919
$^{25}Mg_{12}$	5/2	0.10	0.096197	0.182700	−0.144221
$^{27}Al_{13}$	5/2	≈ 1	−0.409555	0.098408	−0.262229

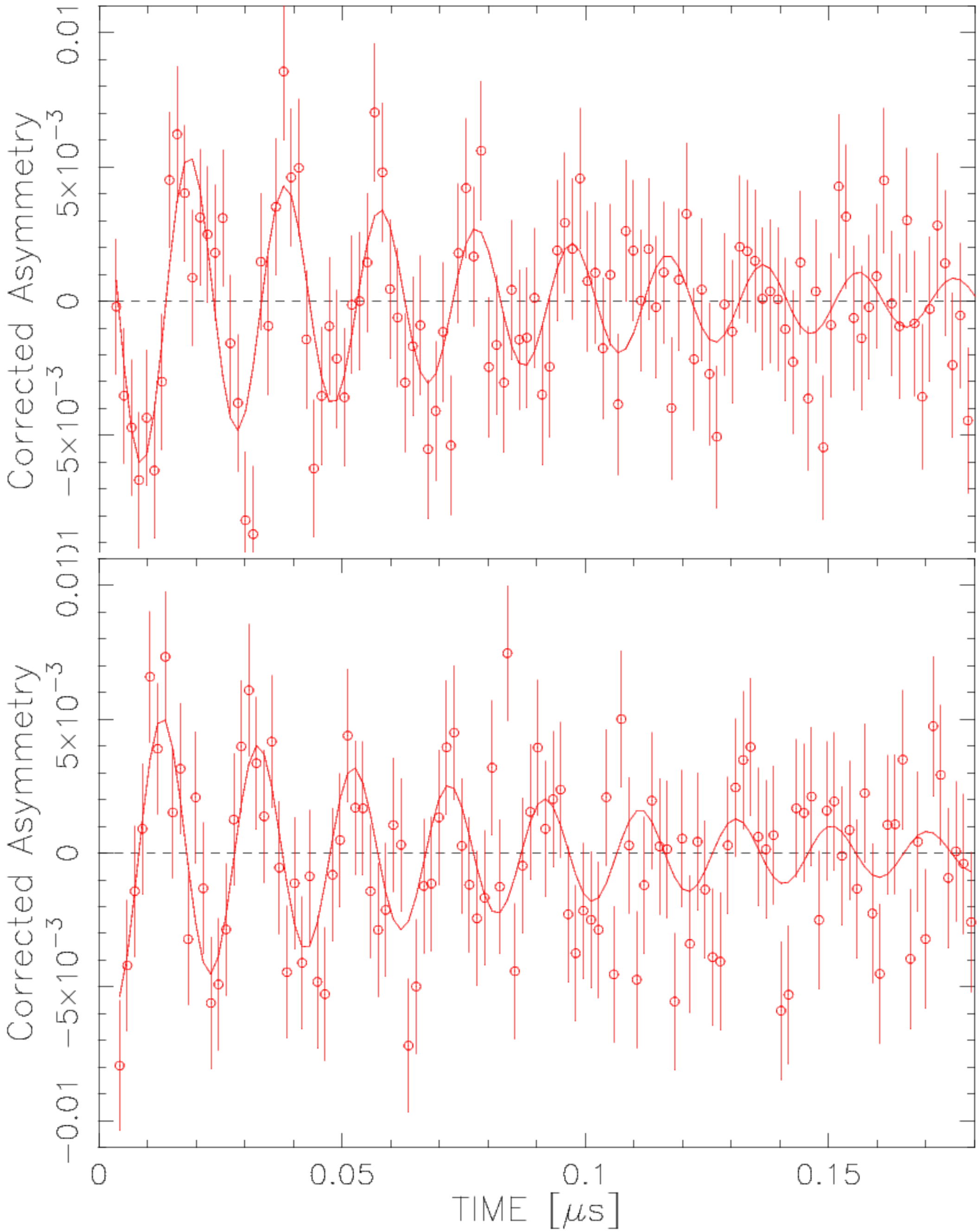
$\mu^{-}SR$ in Teflon $[(CF_2)_n]$:

High frequency signal: $^{12}C\,\mu^{-}$

Low frequency signal:

F^{+} (triplet) state of $^{19}F\mu^{-}$ → HF transition rate $R_{HF} = 5.2(5)\ \mu s^{-1}$ → F^{-} (singlet) state

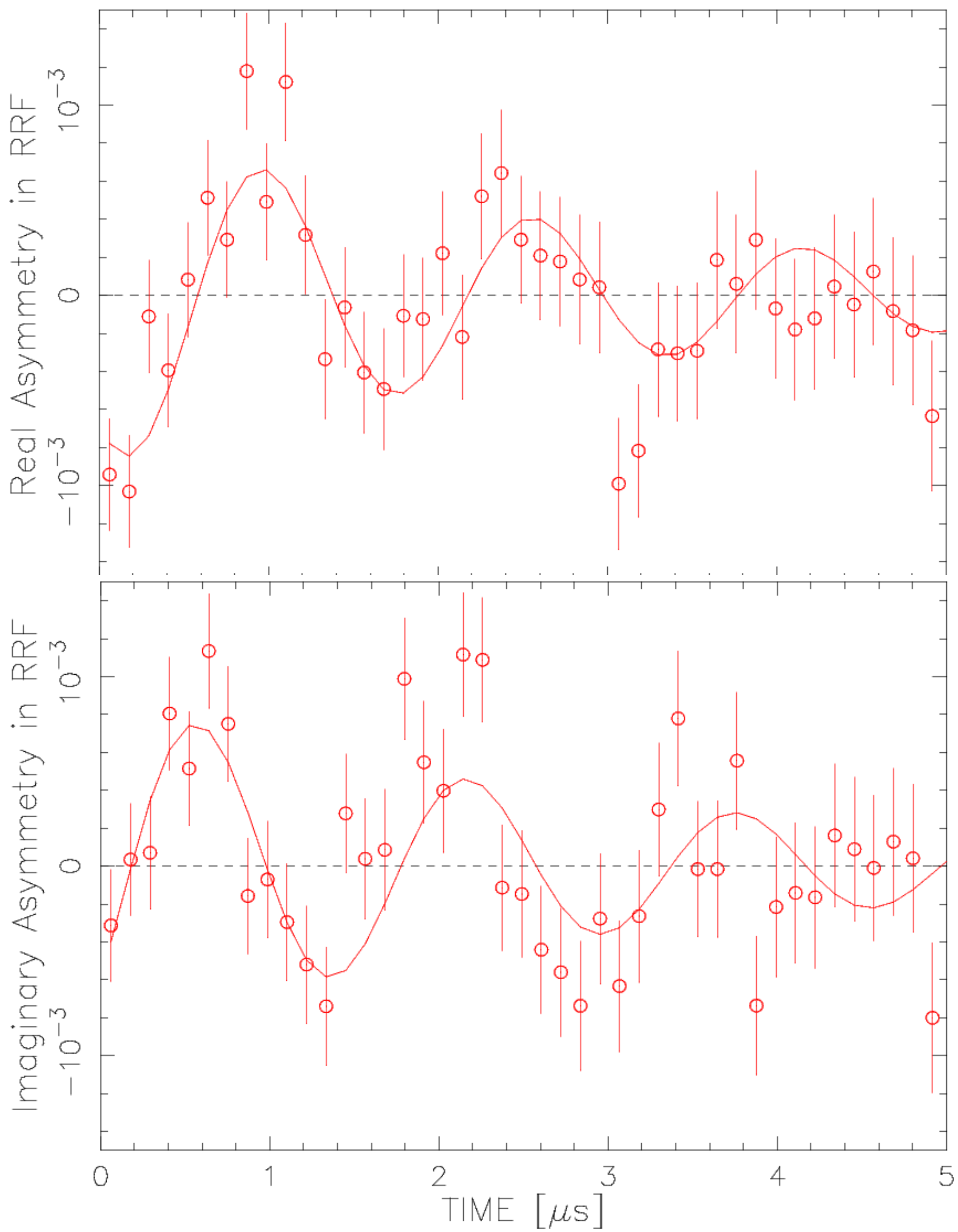




^{23}Na μ^-
F^+ state

$f_+ \bullet P_+(0) = 0.064(8)$

$R_{HF} = 14(3)\ \mu s^{-1}$



^{23}Na μ^-

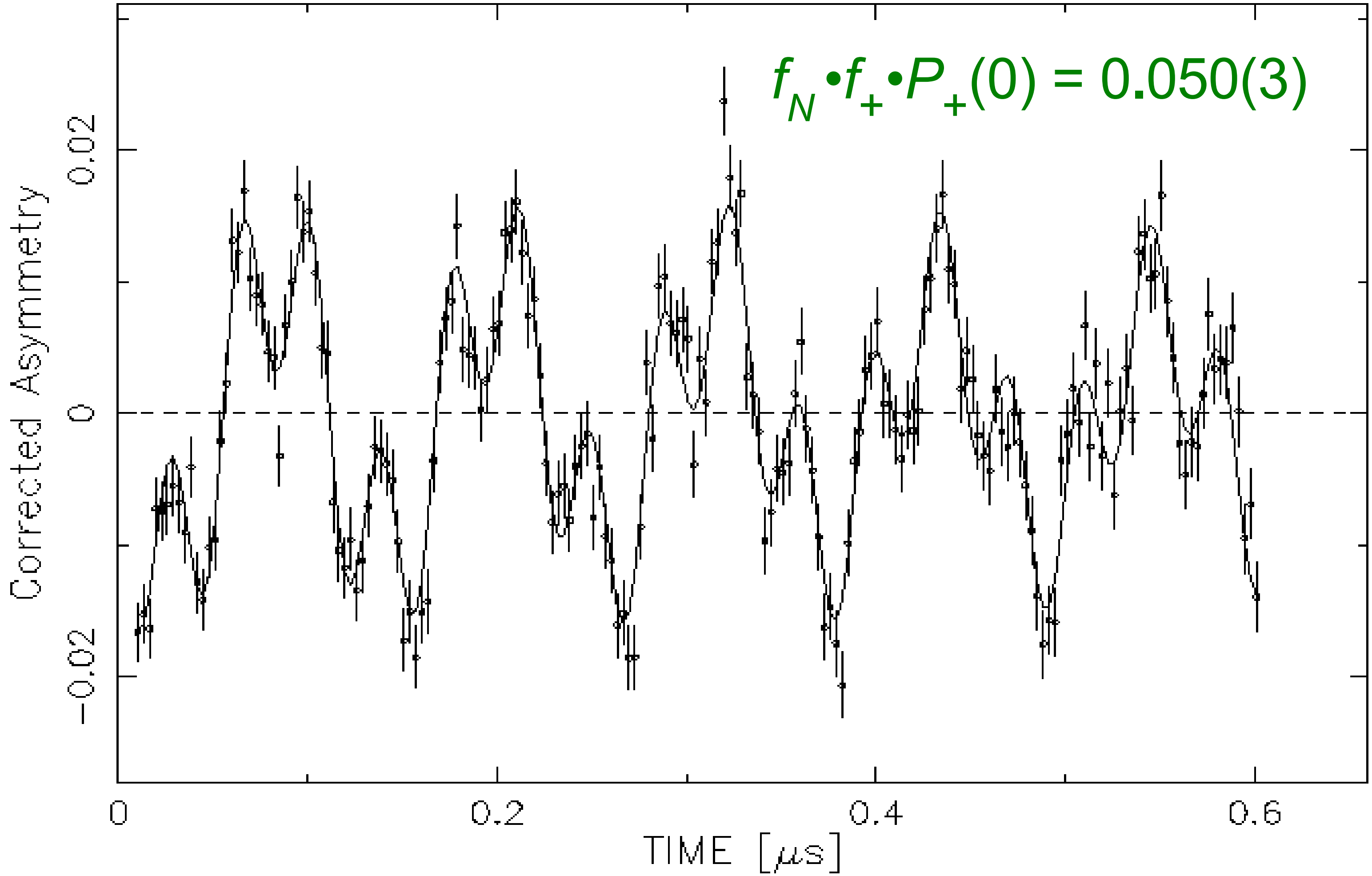
F^- state

$f_- \bullet P_-(0) = 0.0090(15)$

$T_2^{-1} = 0.024(100)\ \mu s^{-1}$

μ^-SR in Melamine ($C_3H_6N_6$):

High frequency signal: $^{12}C\ \mu^-$ Low frequency signal: F^+ state of $^{14}N\ \mu^-$

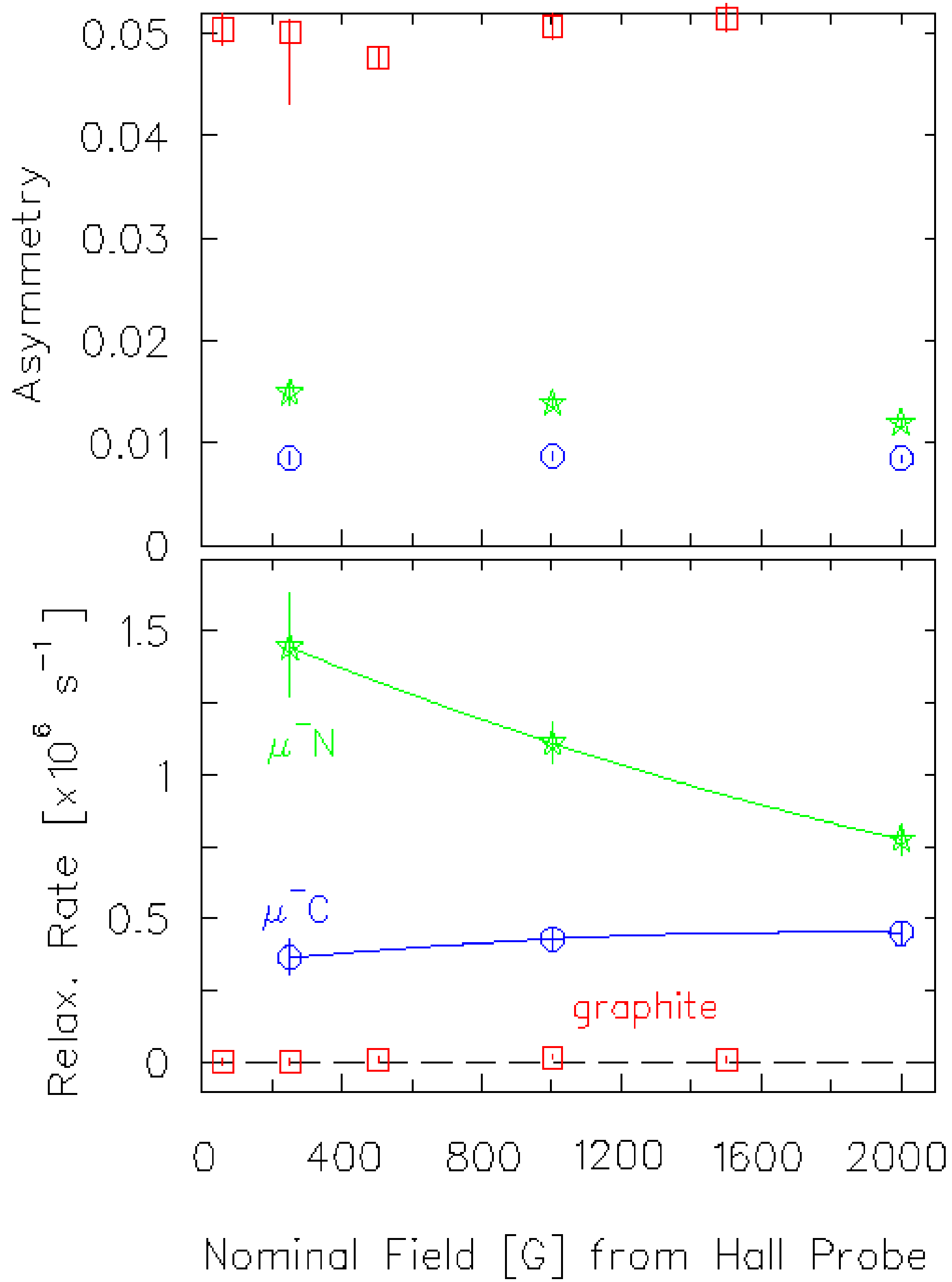


μ^-SR in Melamine ($C_3H_6N_6$)

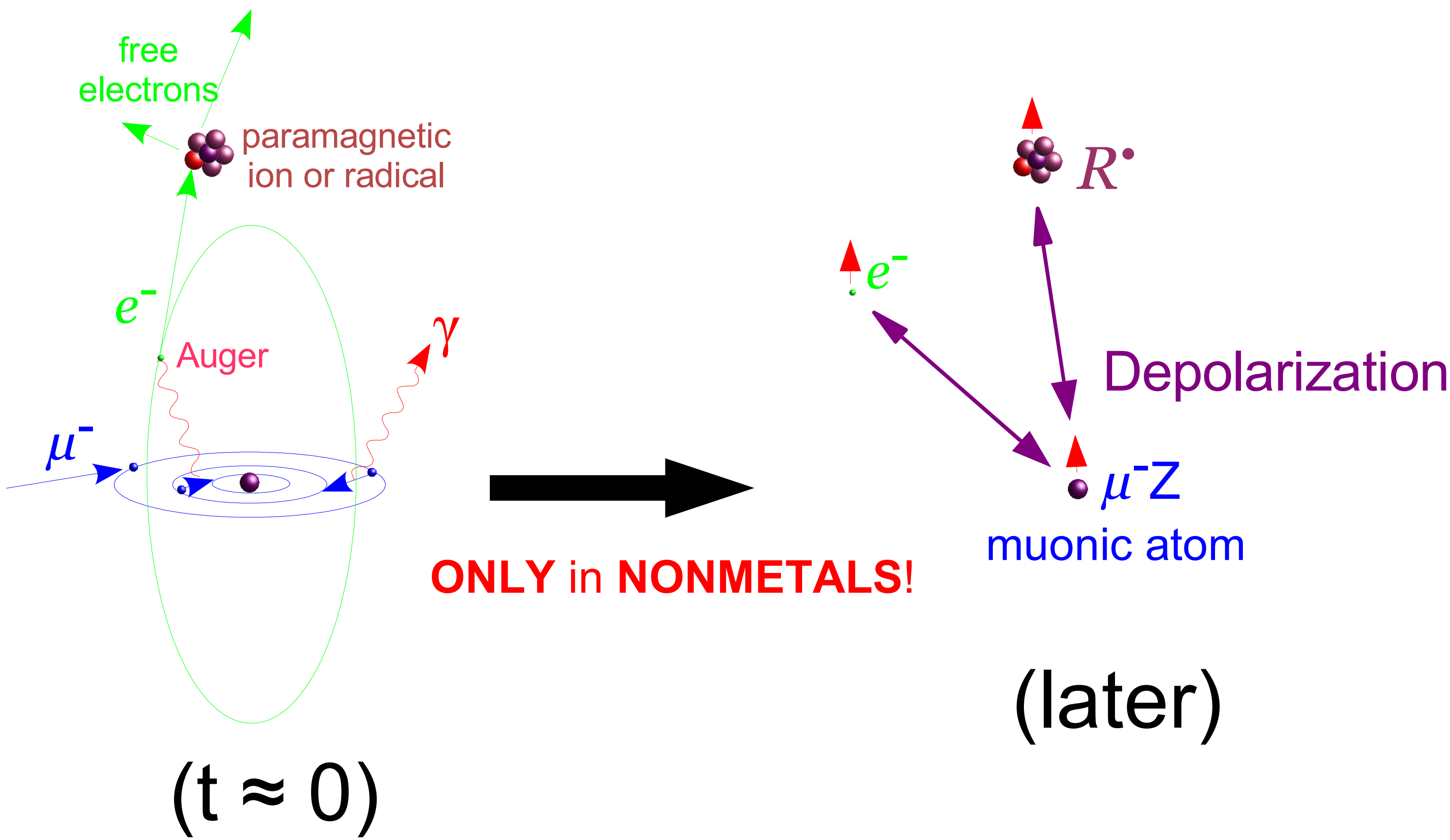
$f_N \bullet f_+ \bullet P_+(0)$ and Λ_N both decrease with ***B***

and Λ_N is much too large to be caused by either R_{HF} or neighbouring nuclear dipoles.

Λ_C is also anomalously fast.



"Coulomb Explosion" Leftovers



Brewer's List of ***μSR*** Acronyms