Quantum Mechanics: Frequently Asked Questions

(original version by Jim Carolan, Nov. 1993)

Wave Mechanics in general

1. Why are we doing this? Because quantum physics explains the microscopic world.

2. What is this wave function Ψ? It is the function which gives all the information that there is about a quantum system. A quantum system can be one or many particles.

3. What is the probability amplitude? That is another word for the wave function Ψ. It is called the probability *amplitude* because the *square* of its absolute value is the probability *density* $W = |\Psi|^2$. This probability W tells us whether the quantum particle is likely to be in a certain state or location. If W is large then the probability is high. If W approaches zero then the probability approaches zero. If we integrate the probability density over all all space, the total probability must be 1. A good reference discussing the wave function and wave packets *etc.* is Ch. 6 in *"The Emperor's New Mind"* by Roger Penrose.

4. What are the applications of Ψ? In principle we can use Ψ to calculate all the properties of any physical system. We can use it to calculate particle position, momentum, energy . . . . We can use it to calculate atomic structure, the behaviour of solids, liquids, gases and plasmas. We can describe elementary particles, lasers, superconductivity *etc.*

5. Is Ψ particular to different types of particles? There are basically two types of particles: *bosons* and *fermions*. We will talk about that in a bit. We don't have to worry about it yet. The spatial part of the wave functions that we are talking about now will be the same for both types of particles. What is different for different particles (that we are talking about now) is their mass and their electric charge. If they have no *mass* (*e.g.* photons), then they have no *dispersion* which means the phase velocity is the same as the group velocity. If they do have mass then they will have some dispersion (see discussion of wave packets and dispersion further on).

6. Is the complex probability anplitude Ψ just a neat trick? No, it's not. It or something like it is necessary. As we said, to represent a free particle of fixed energy (or k), you need a $|\Psi|^2$ that is uniform, that is the probability of finding the particle is the same everywhere. But at the same time that function Ψ must be able to produce wave-like interference effects. The only way anyone knows how to do this is to use the complex probability amplitude function.

7. What is a wave packet? It is a group of waves, each of different wavelength, added together in a way to produce one wave which only shows up in some finite region of space and time. We have then produced a localized wave packet. If we calculate Ψ for this wave packet function, it will be zero everywhere expect one small region of space and time.

8. Is this wave packet real? The wave packet function is not real in the sense that you can't touch it in the classical manner. It is also not real in the sense that we have chosen a particular model to represent the true quantum state. However the quantum state is real. In fact it is the *only* reality! Our representation of the quantum state by a wave packet is to allow us to visualize a little bit what is going on. Our real contact with the quantun state is by measuring $|\Psi|^2$. And of course since it is a probability, sometimes you have to measure it lots of times. We will see, though, that the situation is not always as vague as it appears in the two-slit experiment.

9. What is $a(k)$? When you are adding up waves of different wavelength to make a localized wave packet, $a(k)$ tells you how much of each wavelength ($\lambda = 2\pi/k$) to use.

10. How is a particle composed of many waves? It isn't. De Broglie thought he could make such a thing with his matter waves to explain the wavelike properties of particles. But it didn't work, *because of the probabilistic nature of the interference*. We have learned from Max Born and others that we have to represent a particle by a probability function. This function, *not the particle itself*, we represent sometimes by the wave packet sum.

11. What is dispersion? When we add up waves to form a wavepacket, then dispersion is the change in the shape of the wave packet with time due to the different velocities of the waves contributing to the wave packet.

12. What is this Gaussian business? The Gaussian shaped wavepacket is just one possible wave shape. We chose it to represent a localized but travelling wave packet because it minimises the uncertainty product of Δx and Δk. For the Gaussian, the contribution of a given value of k to the wave packet is proportional to $\exp\left[-\alpha^2(k-k_\circ)^2\right]$ where α and $k_\circ$ are constants. This factor times some constant is the function $a(k)$. Then the wave packet function is just the integral

$$\Psi(x,t) = \int_{-\infty}^{\infty} a(k) e^{i(kx-\omega t)} dk$$

13. What are standlng waves? Two identical waves travelling towards each other interfere in such a way as to produce one wave in which there are nodes that are fixed in position. This is called a *standing wave*. *E.g.*

$$y(x,t) = \cos(kx-\omega t) + \cos(kx+\omega t) = 2\cos\omega t\cos kx$$

For all times the function $y(x,t)$ has nodes at $kx =$ odd multiples of $\pi/2$.

14. How many Schrödinger equations are there? Basically one. But it has levels of complexity, so we start out with one dimension and ignore the time dependence. Then we will add the time, and then (if time permits) we will look at 3 dimensions.

Expectation Values

15. What is the physical definitign of an expectation value? It is the average value that you would expect to get for some observable quantity like x or p if you measured it many times.

16. What are we measuring with the statistical interpretation? We aren't measuring anything. We are giving predictions for what we expect from a measurement.

17. What exactly are fuzzy and sharp observables? A sharp observable is something that you get the same value for every tine you measure it. It has no uncertainty. A fuzzy observable is some quantity for which you will get different results when you make repeated measurements.

For the stationary states of potential wells the energy of the state is a sharp observable, but the position x and the momentum p are fuzzy. Note: this might seem contradictory because $\langle p\rangle = 0$, but $\Delta p \neq 0$. This is discussed below.

18. Why do the methods for finding Δx and Δp differ? The method isn't really different, it only looks that way. This is because we have chosen the Schrödinger representation and in that picture the wave function is defined as a function of space and time (*i.e.* x and t). So then it is easy to find $\langle x\rangle$ and $\langle x^2\rangle$. Finding $\langle p\rangle$ and $\langle p^2\rangle$ is a little harder (see next question), but the method for finding the expectation value is the same. Note that if we chose a momentum - frequency representation instead of a space - time representation then the momentum expectation value would look simple and the position one would look more complicated.

19. Where did the operator come from? For momentum, why is it different from just p? The operator is a function which, when acting on Ψ in the probability integral, produces an expectation value for that quantity In the Schrödinger representation. The operator for x is just x, for x^2 it is just x^2. But for momentum we have to find *a way to write* p *in terms of* x *or* t, so that we can do the integral

$$\langle p\rangle = \int_{-\infty}^{\infty} \hat{p}_{\text{op}}|\Psi|^2 dx = \int_{-\infty}^{\infty} \Psi^* \hat{p}_{\text{op}}\Psi dx$$

The trick we found was to note that

$$\frac{\partial\Psi}{\partial x} = ik\Psi = i\frac{p}{\hbar}\Psi$$

This then says that we can do the integral by equating $\hat{p}_{\text{op}} \equiv -i\hbar\partial/\partial x$, the momentum operator in the Schrödinger representation.

20. If the momentum average iz zero then Δp must be zero. Then shouldn't $\Delta x \to \infty$? No. Just because $\langle p\rangle = 0$ doesn't necessarily mean that $\Delta p = 0$. For the "stationary" states in the well the *magnitude* of the momentum is not on average zero. We know this because we calculated $\langle p^2\rangle$ and found that was not zero. Also remember this is the same as saying the particle has kinetic energy, which it does. The reason $\langle p\rangle = 0$ is because on average the particle is moving to the right the same amount of time it is moving to the left.

Square Potential Wells

21. If $\langle p\rangle = 0$ then why is the probability exponential? See comments about $\langle p\rangle = 0$ above. The fact that the average of the momentum is zero doesn't mean the magnitude of the momentum is zero.

22. If it (the particle) is in the box, then the probability that it is outside should be zero. It is in the box only in the classical sense. Quantum objects can do things classical particles can't do. In the quantum world a particle can be found in places where it is classically impossible for it to exist. However the particle can't *stay* in these classically forbidden regions; it can only "visit".

23. Why shouldn't the probability (at the edge of the well) change abruptly? The amplitude is governed by the Schrödinger equation which describes a wave. The equation says that the second derivative is finite unless the potential becomes infinite. Therefore the function Ψ and its first spatial derivative must be continuous smooth functions.