UBC Physics 401 Lecture:

CAVITY RESONATORS

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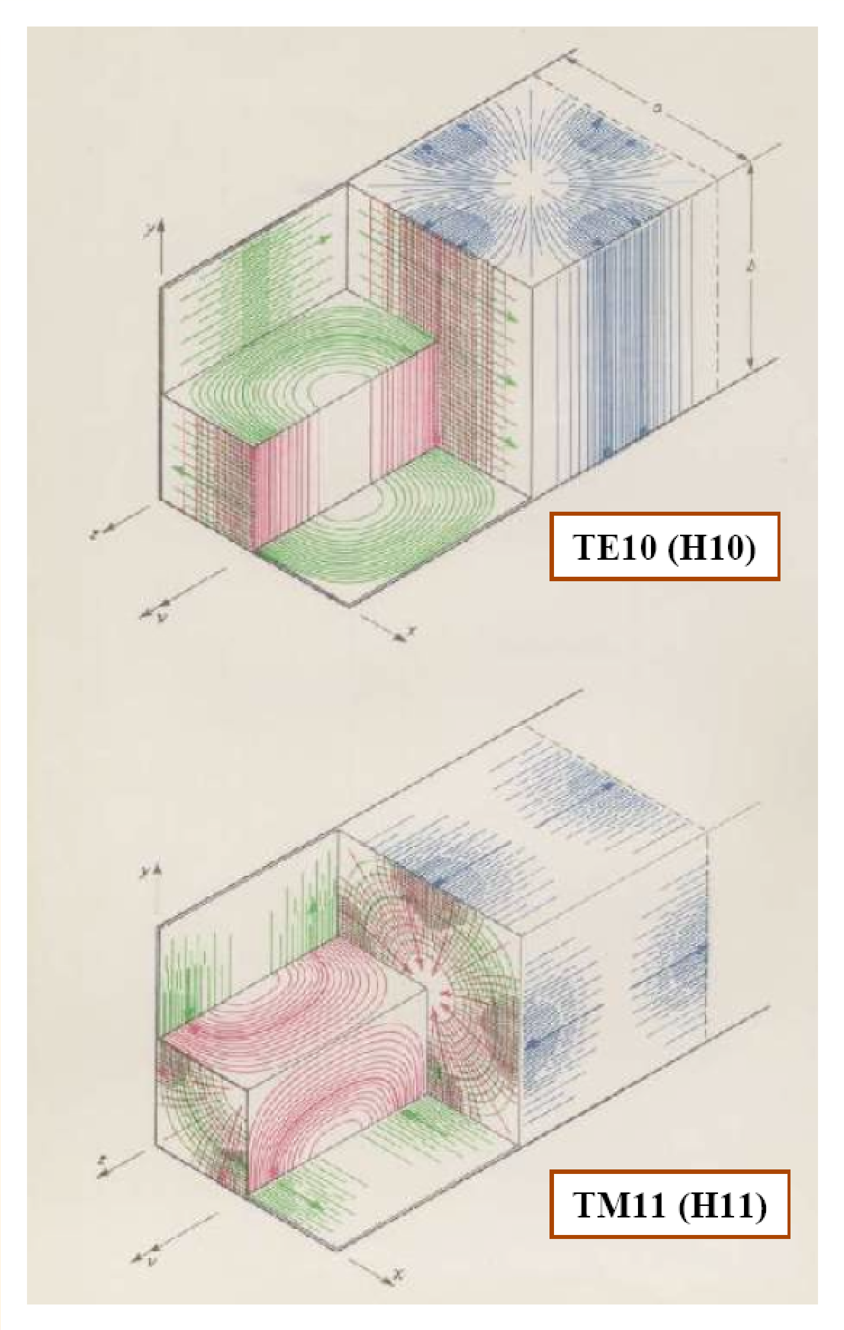
http://musr.physics.ubc.ca/p401/pdf/CavityResonatorsPlus.pdf



We have been studying **rectangular waveguides**. Now it's time to think about what happens when we *close off the ends*. . .

Griffiths relegates this to a Problem, which seems a little glib for such a vitally important technology.

Sketch of instantaneous $\vec{E}$ and $\vec{B}$ fields for TE_{10} and TM_{11} modes in a rectangular waveguide (from Jeffries, *via* Janis).

Rectangular Boxes

Suppose we make a *closed* rectangular box (width a, height b, length d) from a perfect conductor and (somehow) set an EM wave loose inside.[1] We can solve it rather easily by assuming the captured wave is a simple plane wave (phase velocity $v = c = \omega/k$) bouncing around inside, subject only to the constraint that **all ends of the box must be nodes**:

$$k^2 = k_x^2 + k_y^2 + k_z^2, \quad \text{with} \quad k_x = m\frac{\pi}{a},\ k_y = n\frac{\pi}{b},\ k_z = \ell\frac{\pi}{d}\,. \tag{1}$$

For a given **mode** $\{\ell m n\}$ this defines an *unique* frequency

$$\omega_{\ell m n} = ck = c\pi\sqrt{\left(\frac{\ell}{d}\right)^2 + \left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \tag{2}$$

at which the wave can oscillate and still satisfy all the boundary conditions. In other words, we have a **resonant cavity** with various allowed modes and no others.

[1] This is exactly the same problem you have seen in the derivation of *thermal blackbody radiation* in Statistical Mechanics or *particle in a box* in Quantum Mechanics. (In fact, since photons *are* particles, it is *literally* the same problem!)

If we assume $d > a > b$, then the *lowest*-frequency mode will be $\{100\}$ with $\omega_{100} = c\pi/d$.

For a cavity made from an *imperfect* conductor there are small Ohmic losses in the skin depth, reducing the "Q" of the cavity (basically the ratio of the resonant frequency to the decay time of an undriven mode) to a finite value.

Such **cavity resonators** come in different shapes (cylindrical cavities are also important) and are used for everything from microwave probes of superconductivity to accelerators. (In the latter case the cylindrical cavities must have a hole in the ends for particles to go through; this does not prevent the formation of a very stable resonance.)

You may wonder how one "injects" RF power into such a cavity. There is usually a very small "antenna" inserted through one wall which "tickles" the cavity with a weak driving field in the same direction as the resonant field at that location — a typical example of resonance!