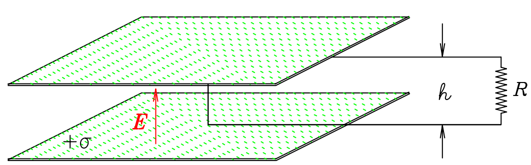
The University of British Columbia

Physics 401 Assignment # 2: Review of Electrodynamics SOLUTIONS:

Wed. 11 Jan. 2006 — finish by Wed. 18 Jan.

Please review Chapter 7. Numbered problems are (as usual) taken from the course textbook: David J. Griffiths, *"Introduction to Electrodynamics"*, 3rd Edition.

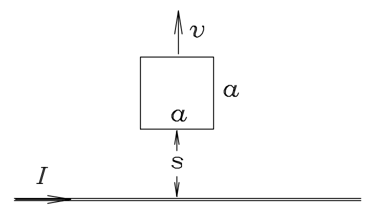
1. (p. 293, Problem **7.6**) — **Understanding EMF:** A rectangular loop of wire is situated so that one end (height h) is between the plates of a parallel-plate capacitor (see figure), oriented parallel to the field $\vec{E}$. The other end is 'way outside, where the field is essentially zero. What is the emf around this loop? If the total resistance is R, what current flows? *Explain.* [*Warning:* this is a trick question, so be careful. If you have invented a perpetual motion machine, there's probably something wrong with it.]



ANSWER: Obviously, if you believe Faraday's Law, there is **no** emf around the loop. In principle this would not exclude a current flowing through R, if there were a battery at the other end to provide the voltage rise to go with its voltage drop. So the question really boils down to whether you can use an electric field as a battery. This sounds silly until you start asking *where the charges go*, at which point it gets confusing enough to raise doubts. So that's what this is really all about. Where *do* the charges go? Well, there are definitely surface charges on the wire (although none on the inside; it is, after all, a conductor, and $\vec{E} = 0$ inside). Positive surface charge will accumulate on the top of the loop and a corresponding amount of negative charge will go to the bottom, inside the capacitor. In fact, the entire potential drop between the capacitor plates will have to occur in the gaps between the wire and the plates, since the wire is an equipotential.[1] So why don't the electrons on the bottom zip around through R to meet up with their positive mates on top and provide a nice perpetual motion machine? Because the same plate's electric field is holding them back! As we move *outside* the capacitor, $\vec{E}$ "bulges" outward, giving a component parallel to the wire that pulls electrons back toward the plate on the bottom and pulls any positive charges back toward the plate on the top. When we get far enough away from the capacitor for $\vec{E}$ to be negligible, the charges are happy to stay where they are. This question is potentially confusing only because it is so simple and we aren't used to thinking in these terms.

[1] If we get too close, the resulting electric fields will be strong enough to cause breakdown, which is why one tries to keep wires out from between high voltage plates. Doh.

2. (p. 300, Problem **7.8**) — **Motional Induction:** A square loop of wire of side a is near a long straight wire which is carrying a current I, as shown in the figure.



(*a*) Find the magnetic field due to the current carrying wire. **ANSWER**: You've calculated this many times from Ampère's law. The field due to the wire is coming straight out of the page (which we'll call the $\hat{\phi}$ direction) normal to the plane of the loop and drops off inversely with r, the distance away from the wire: $\boxed{\vec{B}(r) = \frac{\mu_0 I}{2\pi r}\,\hat{\phi}}$.

(*b*) Find the flux of $\vec{B}$ through the loop. **ANSWER**: The flux Φ_B, like $\vec{B}$, is "up" through the loop. For a given value of s, the flux through the loop is

$$\Phi_B = \frac{\mu_0}{2\pi} I a \int_s^{s+a} \frac{dr}{r} \quad \text{or}$$

$$\boxed{\Phi_B = \frac{\mu_0}{2\pi} I a \ln\left[\frac{s+a}{s}\right]} \; .$$

(*c*) If the loop is pulled directly away from the wire (upwards in the diagram) at speed v, what is the emf generated? **ANSWER**: Since B drops off as $1/r$, the magnitude of the flux through the loop will decrease as we pull it away from the wire. Since $-\frac{d\Phi_B}{dt} = -\frac{d\Phi_B}{ds} \times \frac{ds}{dt} = -\frac{d\Phi}{ds} \times v$, Faraday's law gives

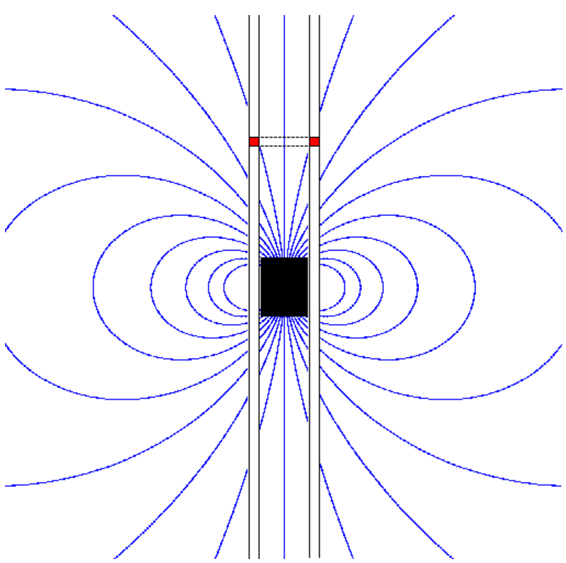
$$\mathcal{E} = -\frac{\mu_0}{2\pi} I \frac{as}{s+a}\left[\frac{1}{s} - \frac{(s+a)}{s}\right] v$$

or $\boxed{\mathcal{E} = -\frac{\mu_0}{2\pi} I \frac{as}{s+a}\left[\frac{1}{s+a} - \frac{1}{s}\right] v}$.

(*d*) Does the induced current flow clockwise or counter-clockwise in the loop?
ANSWER: By LENZ'S LAW, the induced emf around the loop will cause a current to flow that produces its own field in a direction that counteracts the change in flux — in this case, to reestablish the decreasing flux "up" through the loop; that means the current will flow around the loop counterclockwise (as viewed).

(*e*) What is the induced current in the loop if the loop is pulled directly to the *right*, instead of upwards? **ANSWER**: As long as the wire is really semi-infinite, nothing changes (at least not Φ_B) and so there is no emf and no current flows .

3. (p. 305, Problem **7.14**) — **Magnet Falling in Copper Pipe:** As a lecture demonstration, a short cylindrical bar magnet is dropped down a vertical copper tube of slightly larger diameter. It takes several seconds to emerge at the bottom, whereas an otherwise identical piece of *nonmagnetic* iron makes the trip in a fraction of a second. [You have seen this demo at least once.] *Explain why the magnet falls more slowly.* Please don't be glib. Include a diagram of the tube and magnet in your answer/explanation, clearly indicating the directions of any fields, currents or forces.



ANSWER: A truly rigourous calculation is daunting, but we can make a "spherical elephant" approximation that gives the correct qualitative behaviour without bogging down in details or being "glib". Model the tube as a stack of rings. (It isn't, of course, but the main effect does not depend on conduction along the tube axis $\hat{\boldsymbol{z}}$.) Now model the magnet as a simple dipole. Let's assume the North pole is on top so that the magnetic field lines from the magnet point upward along the centre of the tube, as shown. They also, of course, spread out away from the axis, so that as the magnet moves downward, less magnetic flux links a given "ring" of the tube *above* the magnet. This generates an emf in that ring tending to generate an "upward" flux to replace what went away — *i.e.* a current into the page on the right and out of the page on the left. At the position of the ring, $\vec{\boldsymbol{B}}$ has an "outward" horizontal component, which gives the Lorentz force $I d\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{B}}$ (on the induced current in the ring) a downward component all around the ring. NEWTON'S THIRD LAW demands an equal upward force on the magnet due to the current in the ring, but we have recently seen suspicious results from a naive application of said law to magnetic forces, so we should check its veracity with alternate arguments. One would be that the magnetization of the magnet is equivalent to a current ring that goes into the page on the right and out on the left to produce an upward $\vec{\boldsymbol{B}}$ — just like the induced current in the ring! Parallel currents attract, so the magnet is pulled upward by the induced current's field. OK, looks like Newton is safe for now. A ring *below* the magnet sees an *increasing* flux as the magnet approaches, and so "fights the change" with a current in the opposite sense: in on the left and out on the right. This repels the effective current ring of the magnet, so the rings below it also impede its descent. Are we done? Not quite. How come it still falls? If the tube were a perfect conductor, any induced emf would generate a current big enough to completely compensate any attempted change in flux, and a strong enough magnet would simply stop falling. It is only because copper has a finite conductivity that the ring ever comes out the bottom; the "drag" force increases as gravity accelerates the magnet, eventually reaching a terminal velocity determined by the magnet's strength to weight ratio and the conductivity of the tube.

4. (p. 328, Problem **7.35**) — **Coulomb's Law for Magnetic Charges:** [2] Assuming that "Coulomb's Law" for magnetic charges (q_m) reads:

$$\vec{F} = \frac{\mu_0}{4\pi}\,\frac{q_{m_1} q_{m_2}}{\mathcal{R}^2}\,\hat{\boldsymbol{\mathcal{R}}} \qquad \left(\text{where} \quad \vec{\boldsymbol{\mathcal{R}}} \equiv \vec{r} - \vec{r}'\right),$$

work out the analogue of the Lorentz force on a magnetic monopole moving at velocity $\vec{v}$ through electric and magnetic fields $\vec{E}$ and $\vec{B}$.

ANSWER: The whole idea of magnetic monopoles is to completely "symmetrize" MAXWELL'S EQUATIONS with respect to the transformation

$$\begin{array}{lcllcl} \vec{E} & \to & \vec{B} & \vec{B} & \to & -\mu_0\epsilon_0\,\vec{E} \\ \rho_e & \to & \mu_0\epsilon_0\,\rho_m & \rho_m & \to & -\rho_e \\ \vec{J}_e & \to & \mu_0\epsilon_0\,\vec{J}_m & \vec{J}_m & \to & -\vec{J}_e \end{array}$$

(see section **7.3.4** on p. 327). In this spirit, the LORENTZ FORCE $\vec{F}_e = q_e\left(\vec{E} + \vec{v}\times\vec{B}\right)$ transforms into

$$\boxed{\vec{F}_m = q_m\left(\vec{B} - \mu_0\epsilon_0\,\vec{v}\times\vec{E}\right)}\,.$$

(If you like you can substitute $1/c^2$ for $\mu_0\epsilon_0$.)

5. (p. 328, Problem **7.36**) — **Monopole Through Loop:** [3] Suppose a magnetic monopole q_m passes through a resistanceless loop of wire with self-inductance L. *What current is induced in the loop?*

ANSWER: We mean "normal" (electrical) current, of course. Such current is driven by a "normal" (electrical) emf, $\oint \vec{E}\cdot d\vec{\ell}$ around the loop, and that emf is generated by a changing "normal" (magnetic) flux $\Phi_m = \iint \vec{B}\cdot d\vec{a}$: $\mathcal{E} = -\frac{\partial \Phi_m}{\partial t} = -L\frac{\partial I}{\partial t}$. If the current in the loop is initially zero, integrating over time gives $\Delta\Phi_m = L\Delta I$ or $\Delta I = \Delta\Phi_m/L$. So what is $\Delta\Phi_m$? Well, initially the monopole is so far away that $\Phi_m = 0$; as it approaches the loop from (let's say) $x = -\infty$, more and more field lines link the loop until, when it reaches the exact centre of the loop, exactly half the total magnetic flux "emitted" by q_m is coming out the $+x$ side. As it continues on past the loop, even more of its emitted flux is "on that side", until as $x \to +\infty$ the last few field lines are "on the right side" of the loop. That is to say, *all* the flux emitted by q_m has passed through the loop: $\Delta\Phi_m = \oiint \vec{B}\cdot d\vec{a} = B(r)\times(4\pi r^2) = \frac{\mu_0}{4\pi}\,\frac{q_m}{r^2}\times(4\pi r^2)$ or $\Delta\Phi_m = \mu_0\,q_m$, giving

$$\boxed{\Delta I = \Delta\Phi_m/L = \mu_0\,q_m/L}\,.$$

This makes a nice distinctive signal and tells one the magnitude of the detected monopole as well. Note that the size of the loop doesn't matter; monopole detectors can therefore employ lots and lots of very small loops that don't pick up much noise from randomly fluctuating ordinary magnetic fields.

6. **Seminar Topic:** *Write down your proposed seminar topic.*

There's no "correct answer" to this one!

[2] [For an interesting commentary, see W. Rindler, *Am. J. Phys.* **57**, 993 (1989).]

[3] [This is one of the methods used to search for magnetic monopoles in the laboratory; see B. Cabrera, *Phys. Rev. Lett.* **48**, 1378 (1982).]