THE UNIVERSITY OF BRITISH COLUMBIA

Physics 401 Assignment # 11: RADIATION *1*

Wed. 22 Mar. 2006 — finish by Wed. 29 Mar.

1. (p. 449, Problem **11.2**) — **Electric Dipolar Radiation:**

$$\text{Equation (11.14)}, \qquad V(r,\theta,t) = -\frac{p_0\omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin\left[\omega\left(t-\frac{r}{c}\right)\right],$$

can be expressed in "coordinate-free" form by writing $p_0\cos\theta = \vec{\boldsymbol{p}}_0\cdot\hat{\boldsymbol{r}}$. Do so, and similarly for Equations

$$(11.17), \qquad \vec{\boldsymbol{A}}(r,\theta,t) = -\frac{\mu_0 p_0\omega}{4\pi r}\sin\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{z}},$$

$$(11.18), \qquad \vec{\boldsymbol{E}}(r,\theta,t) = -\frac{\mu_0 p_0\omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{\theta}},$$

$$(11.19), \qquad \vec{\boldsymbol{B}}(r,\theta,t) = -\frac{\mu_0 p_0\omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{\phi}},$$

$$\text{and (11.21)}, \qquad \langle\vec{\boldsymbol{S}}\rangle = \left(\frac{\mu_0 p_0^2\omega^4}{32\pi^2 c}\right)\frac{\sin^2\theta}{r^2}\hat{\boldsymbol{r}}.$$

2. **Atomic Dipoles:** Explain why you can safely assume $\dfrac{\vec{\boldsymbol{m}}_0}{c} \ll \vec{\boldsymbol{p}}_0$ for an atom with magnetic dipole moment $\vec{\boldsymbol{m}}_0$ and electric dipole moment $\vec{\boldsymbol{p}}_0$, assuming typical values of relevant physical quantities.

3. (p. 473-474, Problem **11.22**) — **Broadcasting KRUD:** A radio tower rises to a height h above flat horizontal ground. At the top is a magnetic dipole antenna of radius b, with its axis vertical. FM station KRUD broadcasts from this antenna at angular frequency ω, with a total radiated power P (averaged, of course, over a full cycle). Neighbors have complained about problems they attribute to excessive radiation from the tower — interference with their stereo systems, mechanical garage doors opening and closing mysteriously, and a variety of suspicious medical problems. But the city engineer who measured the radiation at the base of the tower found it to be well below the accepted standard. You have been hired by the Neighborhood Association to assess the engineer's report.

(a) In terms of the variables given (not all of which may be relevant, of course) find the formula for the intensity of the radiation at ground level, a distance R away from the base of the tower. You may assume that $b \ll c/\omega \ll h$. [*Note:* we are interested only in the *magnitude* of the radiation, not in its *direction* — when measurements are taken, the detector will be aimed directly at the antenna.]

(b) How far from the base of the tower *should* the engineer have made the measurement? What is the formula for the intensity at this location?

(c) KRUD's actual power output is 35 kilowatts, its frequency is 90 MHz, the antenna's radius is 6 cm, and the height of the tower is 200 m. The city's radio-emission limit is 200 microwatts/cm^2. Is KRUD in compliance?

4. (p. 474, Problem **11.23**) — **Earth as a Pulsar:** The magnetic north pole of the Earth does not coincide with the geographic North Pole — in fact, it's off by about $7°$ at present.[1] Relative to the fixed axis of rotation, therefore, the magnetic dipole moment vector of the Earth is changing with time, so the Earth must be giving off magnetic dipole radiation.

(*a*) Find the formula for the total power radiated, in terms of the following parameters: Ψ (the angle between the geographic and magnetic north poles), M (the magnitude of the Earth's magnetic dipole moment), and ω (the angular velocity of rotation of the Earth). [*Hint:* refer to Prob. 11.4 or Prob. 11.12.]

(*b*) Using the fact that the Earth's magnetic field is about half a gauss at the Equator, estimate the magnetic dipole moment M of the Earth.

(*c*) Find the power radiated. [Your answer should be several times 10^{-5} W.]

(*d*) Pulsars are thought to be rotating neutron stars, with a typical radius of about $R \sim 10$ km, a typical surface magnetic field of $B(R) \sim 10^8$ T and a variety of rotational periods T; let's use $T \sim 10^{-3}$ s. What sort of radiated power would you expect from such a star? [See J.P. Ostriker and J.E. Gunn, *Astrophys. J.* **157**, 1395 (1969). *Answer:* 2×10^{36} W.]

[1] The disagreement between the current value and that in Griffiths is due to the fact that magnetic north pole (which is actually a **south** magnetic pole, of course) has been drifting approximately northwest at about 40 km per year for the last few years (a blistering pace on a geological time scale); it has always wandered around like this, and has reversed direction more than once! Sailors (and students in Power Squadron courses) must learn how to correct their compass readings for this gradual drift.