The University of British Columbia

Physics 401 Assignment # 9: LOOSE ENDS & COAX CABLES

Wed. 8 Mar. 2006 — finish by Wed. 15 Mar.

1. **Cutoff Frequencies:** *Explain in words* why there is a *lower* limit on the frequencies of EM waves that will propagate freely either through a *tenuous plasma* or down a *rectangular waveguide*. Why is there *no* cutoff frequency (neither upper nor lower) for wave propagation down an ideal *coaxial cable*?

2. **Rectangular Waveguide with Dielectric:** Show that if a hollow rectangular waveguide of the type shown in Griffiths Figure 9.24 is completely filled with a dielectric of permeability ϵ, its cut-off frequency is *lower* than if it were empty, by a factor of $\sqrt{\epsilon_0/\epsilon}$:

$$\frac{\omega_{mn}^{\text{dielectric}}}{\omega_{mn}^{\text{vacuum}}} = \sqrt{\frac{\epsilon_0}{\epsilon}}\ .$$

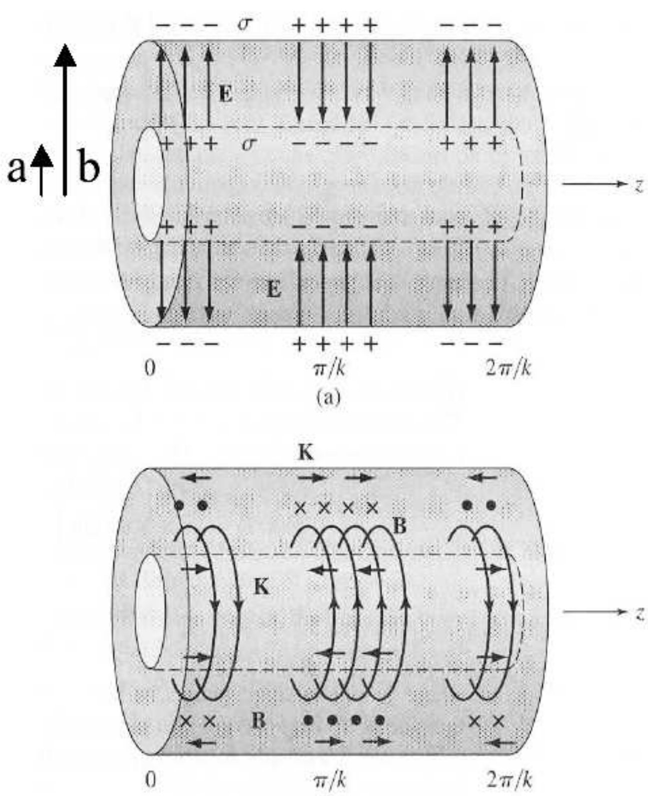
So, for a given operating frequency, a dielectric filled waveguide can be *smaller* than an empty one.

3. (p. 412, Problem **9.31**) — **Coax Cable:**[1]

(a) Show directly that Eqs. (9.197) satisfy Maxwell's equations (9.177) and the boundary conditions (9.175).

(b) Find the net charge per unit length, $\lambda(z,t)$, and the net current, $I(z,t)$, on the inner conductor.

4. **Coax Impedance:**



In class, we derived the electric and magnetic fields in a coaxial transmission line. From those we deduced the characteristic impedance of a coaxial cable:

$$Z = \frac{V(z,t)}{I(z,t)} = \frac{\ln(b/a)}{2\pi}\sqrt{\frac{\mu}{\epsilon}} = 60\ \Omega \cdot \ln(b/a)$$

where a is the radius of the inner coax line and b is the radius of the outer coax cylinder, as shown.

In general, the characteristic impedance of a transmission line is given by

$Z = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}$, where $\mathcal{L}$ and $\mathcal{C}$ are the inductance and capacitance per unit length, respectively.

Show that the characteristic impedance of this coax line satisfies this definition by calculating $\mathcal{L}$ and $\mathcal{C}$ explicitly, and then Z.

[1] *Hint:* in part (b) you will find the charge density on the inner cylinder is $\lambda = 2\pi\epsilon_0 E_0 \cos(kz - \omega t)$.