THE UNIVERSITY OF BRITISH COLUMBIA

Physics 401 Assignment # 3: CONSERVATION LAWS

Wed. 18 Jan. 2006 — finish by Wed. 25 Jan.

1. (p. 340, Problem **7.58**) — **TRANSMISSION LINE:** A transmission line is constructed two parallel thin metal "ribbons" of width w separated by a very small distance $h \ll w$. The current travels down one strip and back along the other. In each case it spreads out uniformly over the surface of the ribbon.

(a) Find the capacitance per unit length, $\mathcal{C}$.

(b) Find the inductance per unit length, $\mathcal{L}$.

(c) What is the product, $\mathcal{LC}$, numerically? [1]

(d) If the strips are insulated from one another by a nonconducting material of permittivity ϵ and permeability μ, what is then the product $\mathcal{LC}$? What is the propagation speed? [2]

2. (p. 349, Problem **8.1**) — **POWER TRANSMISSION:** Calculate the power (energy per unit time) transported down the cables of Exercise 7.13 (p. 319) and Problem 7.58 (p. 340), assuming the two conductors are held at a potential difference V, and carry current I (down one and back up the other).

3. (p. 357, Problem **8.5**) — **FORCE on a PARALLEL PLATE CAPACITOR:** Consider a semi-infinite parallel plate capacitor (far from the edges), with the lower plate (at $z = -d/2$) carrying a uniform charge density $-\sigma$ and the upper plate (at $z = +d/2$) carrying a uniform charge density $+\sigma$.

(a) Determine all nine elements of the stress tensor in the region between the plates. Display your answer as a 3×3 matrix:

$$\begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$$

(b) Use Eq. (8.22) on p. 353 to determine the force per unit area on the top plate. Compare Eq. (2.51) on p. 103.

(c) What is the momentum per unit area, per unit time, crossing the xy plane (or any other plane parallel to that one, between the plates)?

(d) At the plates this momentum is absorbed, and the plates recoil (unless there is some other force holding them in position). Find the recoil force per unit area on the top plate, and compare your answer to that in part (b). [3]

4. (p. 361, Problem **8.9**) — **SOLENOID and RING:** A very long solenoid of radius a, with n turns per unit length, carries a current I_S. Coaxial with the solenoid, at radius $b \gg a$, is a circular ring of wire with resistance R. When the current in the solenoid is gradually decreased, a current I_r is induced in the ring.

(a) Calculate I_r in terms of dI_S/dt.

(b) The power ($I_r^2 R$) delivered to the ring must have come from the solenoid. Confirm this by calculating the Poynting vector just outside the solenoid, where the *electric* field is due to the changing flux in the solenoid and the *magnetic* field is due to the current in the ring. Integrate over the entire surface of the solenoid, and check that you recover the correct total power.

[1] $\mathcal{L}$ and $\mathcal{C}$ will, of course, vary from one kind of transmission line to another, but their *product* is a universal constant — check, for example, the cable in Exercise 7.13 on p. 319 — provided the space between the conductors is a vacuum. In the theory of transmission lines, this product is related to the speed at which a pulse propagates down the line ($v = 1/\sqrt{\mathcal{LC}}$).

[2] *Hint:* see Exercise 4.6 on p. 183; by what factor does $\mathcal{L}$ change when an inductor is immersed in linear material of permeability μ?

[3] *Note:* this is not an *additional* force, but rather an alternative way of calculating the *same* force — in (b) we got it from the force law, and in (d) we got it from conservation of momentum.

5. PHOTON DRIVE: Rocket ships propelled by photon drives often appear in science fiction novels and movies. The idea is to generate thrust by expelling photons. Since the "exhaust velocity" of photons is as high as you can get (the speed of light), you might expect photon drive rockets to outperform conventional rockets.

(a) Calculate the *power* you'd need to produce 1 Newton of thrust with a photon drive rocket. How does this compare with the typical output of BC's huge Stave Lake power station, which has a peak capacity of about 200 MW?

(b) What accelerations would result if your power source provided 200 MW and the total mass of the rocket were 20,000 kg?

(c) In spite of these numbers, the photon drive offers one very attractive advantage over conventional rockets, especially for *long* space voyages. What is it?