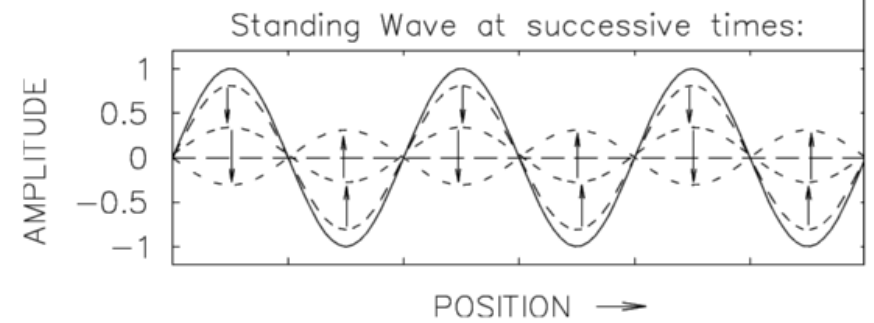
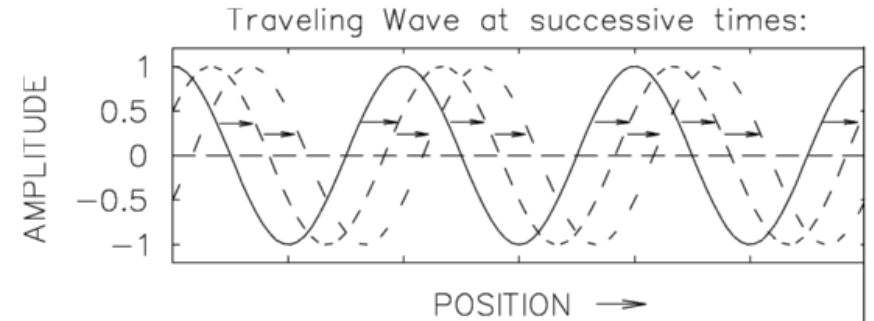
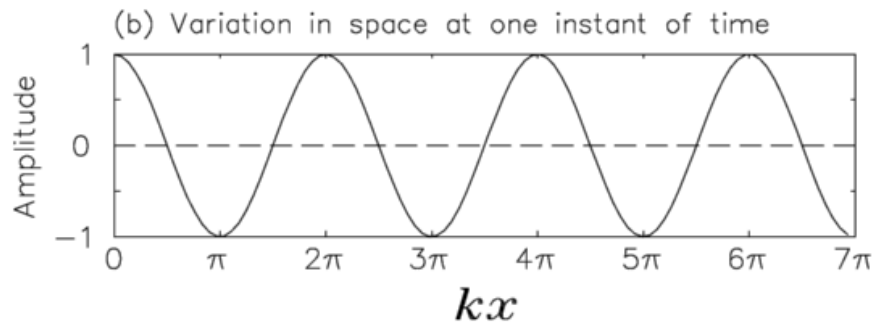
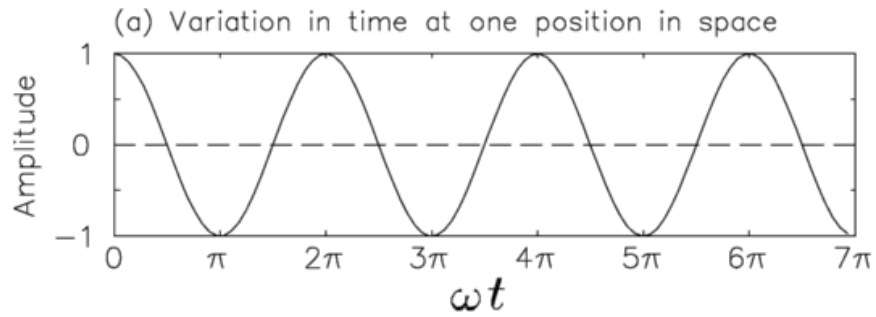


# WAVES



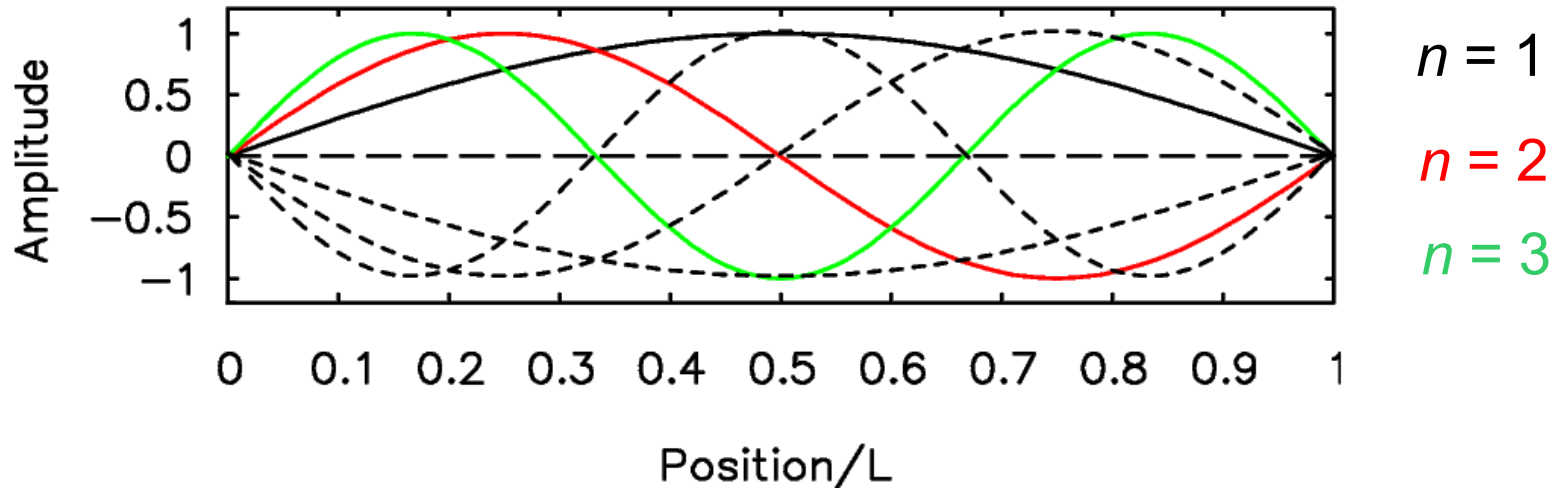
$$\omega = 2\pi/T$$

$$k = 2\pi/\lambda$$

$$\omega/k = \lambda v = c$$

# MODES

of  
Standing Waves



If the ends of a string are held fixed, the only allowed **modes** of vibration are those with an integer number of half-wavelengths in  $L$ :  $L = n (\lambda_n/2)$

or

$$\lambda_n = 2L/n$$

# De Broglie's Hypothesis

Planck and (in 1905) Einstein had explained that waves (light) propagate as quanta ("photons") of energy  $\epsilon = h \nu$  where  $\nu$  is the frequency of the wave's oscillation and  $h = 6.626 \cdot 10^{-34}$  J-s is Planck's constant.

It was also well known that the energy of such a wave is equal to its momentum times its speed of propagation:  $\epsilon = p c$ . For any wave,  $c = \lambda \nu$ , and so, for photons,  $p = h/\lambda$ .

Louis de Broglie simply postulated that if light acts like particles, maybe electrons (or any particles) also behave like waves, and in particular have wavelengths given by

$$\lambda = h/p.$$

# Bead on a Frictionless String

$$\lambda_n = 2L/n$$

$$\lambda = h/p$$

$$p_n = h/\lambda_n = nh/2L$$

Nonrelativistically,  $p = m v = m 2L/\Delta t$  so  $\Delta t_n = 2Lm/p_n = 4L^2m/nh$

giving  $E_n = p_n^2/2m = n^2 h^2/8m L^2$

and  $F_n = 2p_n/\Delta t = n^2 h^2/4m L^3$

# Relativistically...

Total Relativistic Energy:  $E_n^2 = p_n^2 c^2 + m^2 c^4$  (Always true!)

For  $E_n \gg m c^2$  ("ultrarelativistic") we have  $E_n \approx p_n c$  and if we take  $m_{\text{eff}} = E/c^2$  and  $n = 1$ , then  $m_{\text{eff}} \approx h/2Lc$ .

An object in a circular orbit around a mass  $M$  has an orbital velocity

$v_{\text{orb}} = \sqrt{\frac{GM}{R}}$ . If  $v_{\text{orb}} = c$ , then even light can't escape beyond the

Schwarzschild radius,  $R_S = \frac{GM}{c^2}$ . What does it take to get  $R_S = L$ ?

$$L = \frac{Gm_{\text{eff}}}{c^2} = \frac{Gh}{2Lc^3} \quad \text{or}$$

$$L_{\text{Planck}} = \sqrt{\frac{Gh}{2c^3}}.$$

If even a photon is confined to this small a region, the effect of the confinement will raise its energy enough to make it a **black hole!**

(Numbers are off by small factors like 2 or  $\pi$ .)