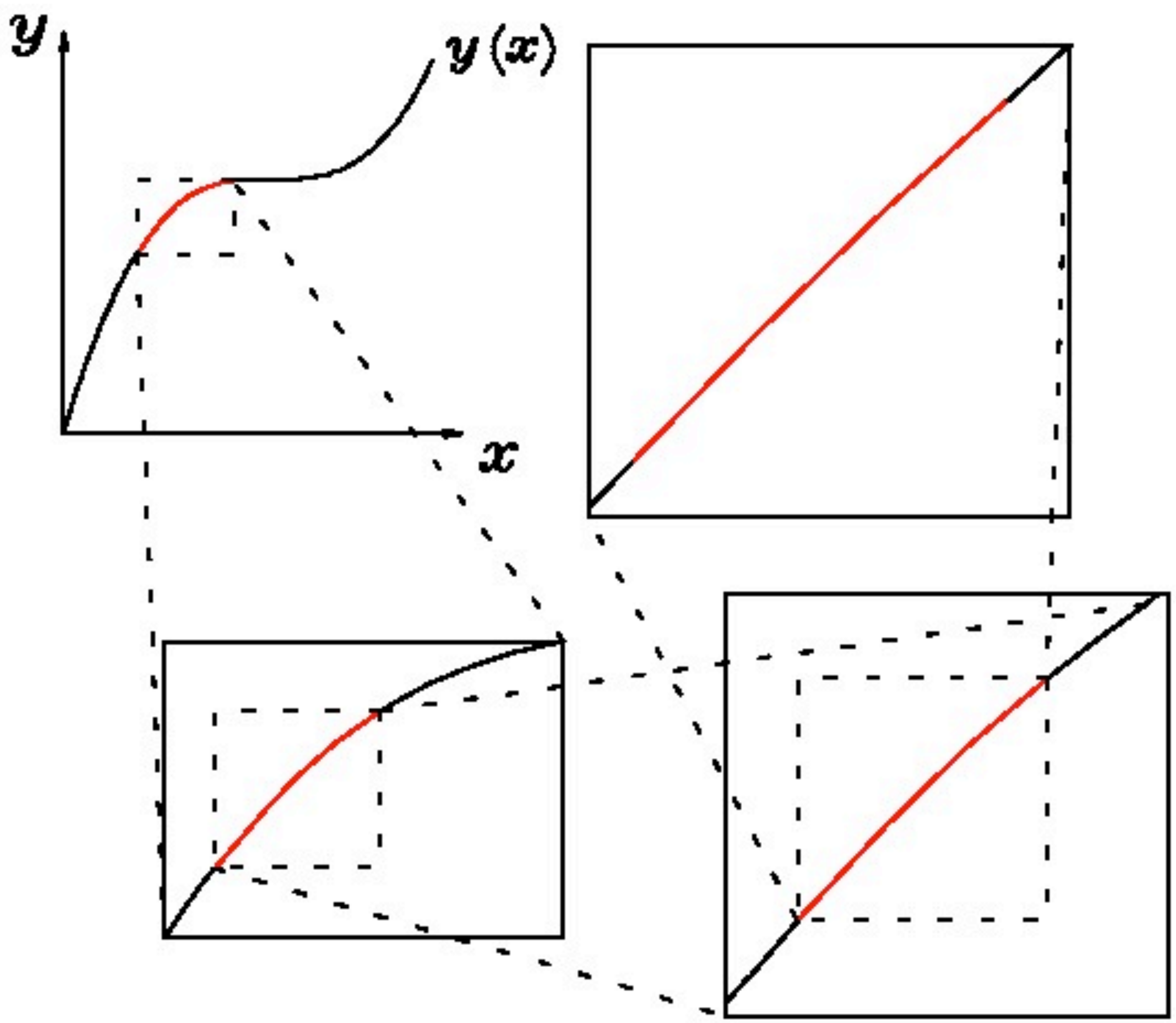
Math* for *Physicists

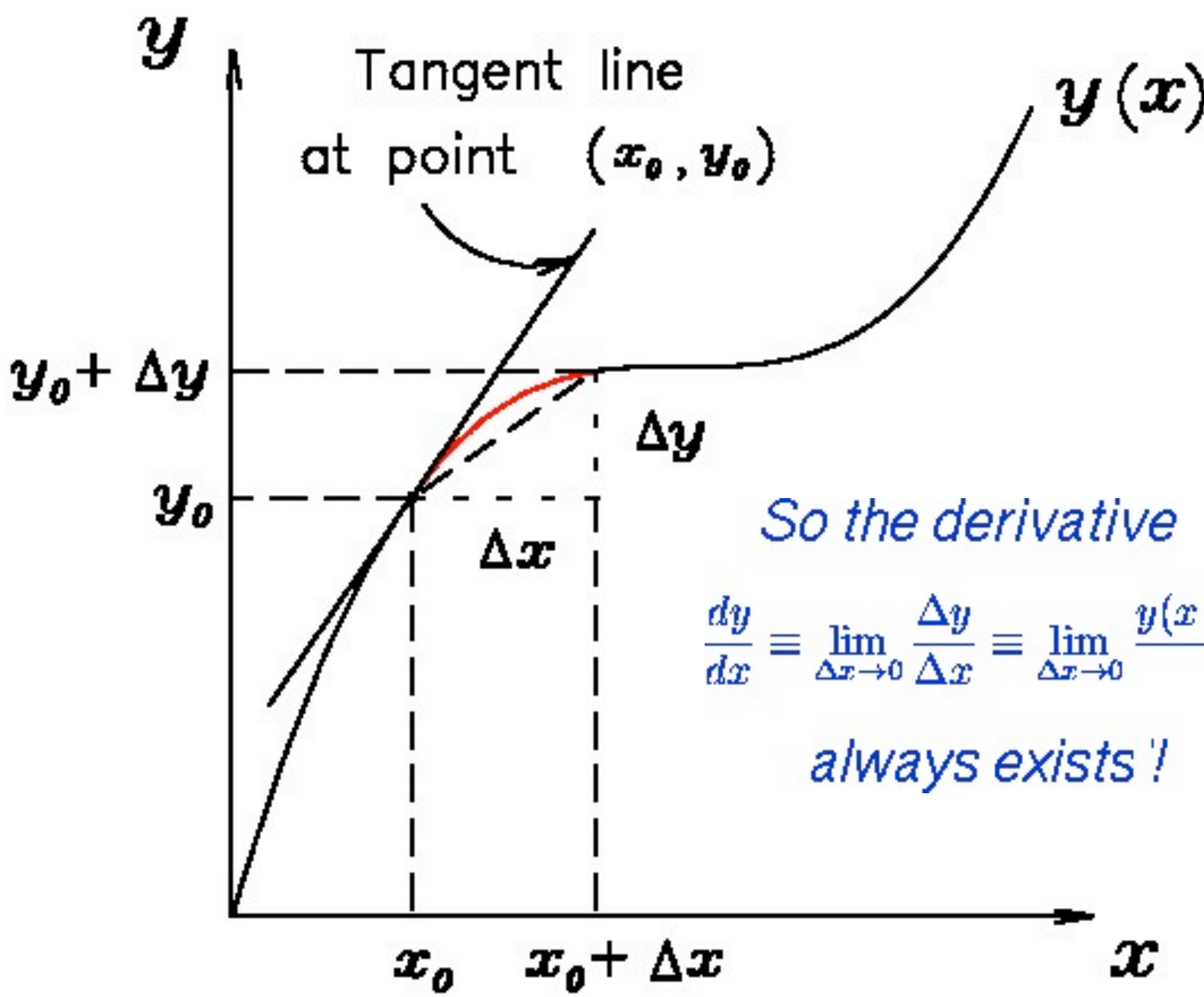
a Hand-Waver's Guide to Calculus

(blame ***Jess***)

Rule 1: A *curved* line looks *straight* if you *blow it up* enough!



Rule 2: There are ^(probably) *no discontinuities* in the real, physical world.



So the derivative

$$\frac{dy}{dx} \equiv \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \equiv \lim_{\Delta x \to 0} \frac{y(x + \Delta x) - y(x)}{\Delta x}$$

always exists !

A few easy-to-remember derivatives:

Power Law:

$$\frac{d}{dx}(x^p) = p\,x^{p-1}$$

$(p \neq 0)$

Constant × a Function:

$$\frac{d}{dx}\left[a\,y(x)\right] = a\,\frac{dy}{dx}$$

$(a = \text{const})$

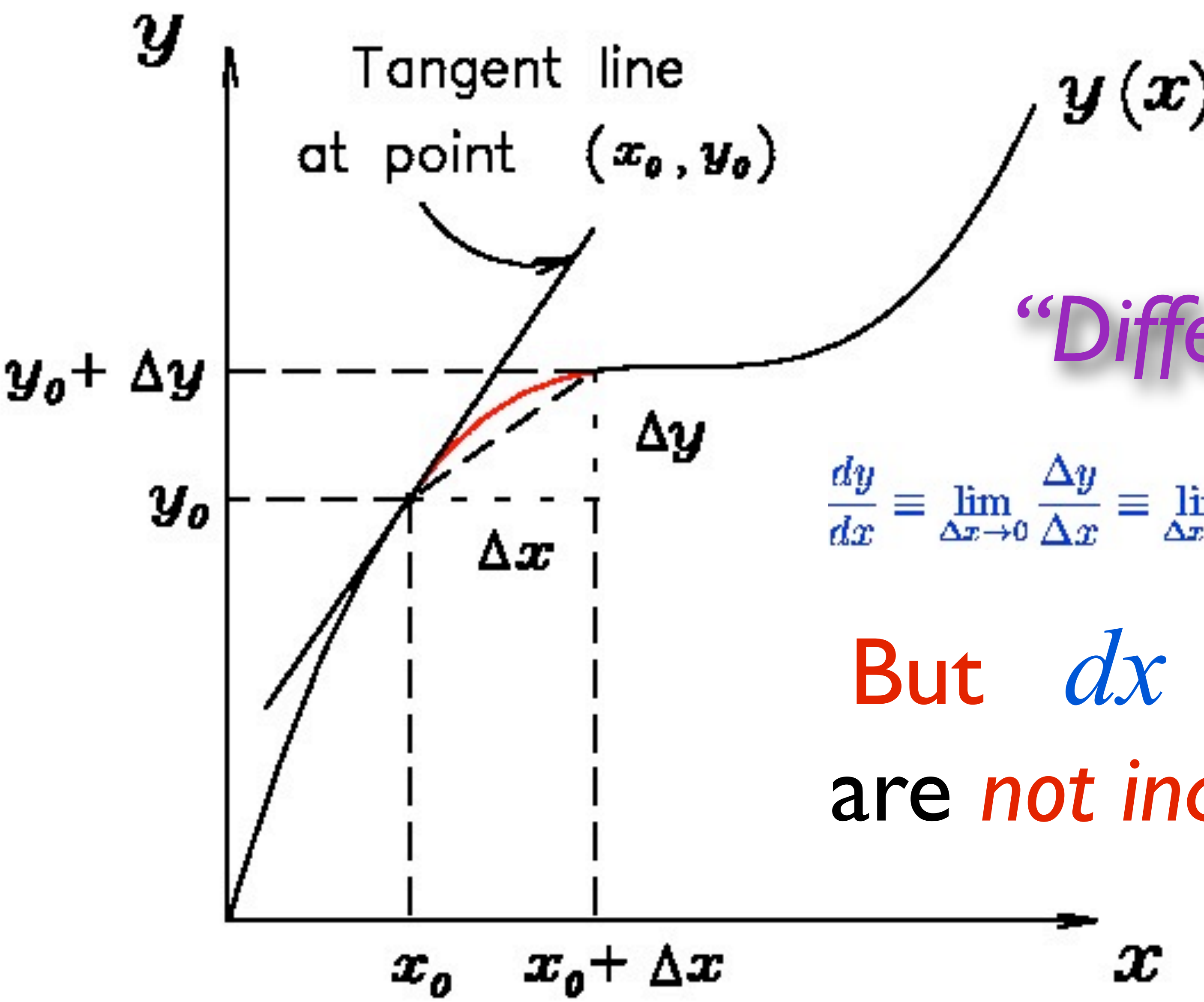
Product Law:

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right] = \frac{df}{dx}\cdot g(x) + f(x)\cdot\frac{dg}{dx}$$

Chain Rule:

$$\frac{d}{dt}\,y[x(t)] = \frac{dy}{dx}\cdot\frac{dx}{dt}$$

Rule 3: dx is just a really, *really small* Δx



"Differentials"

$$\frac{dy}{dx} \equiv \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \equiv \lim_{\Delta x \to 0} \frac{y(x+\Delta x) - y(x)}{\Delta x}$$

But dx and dy are *not independent!*

Rule 4: If we're really, *really careful* and *never forget* that dv, dx and dt are *not independent*,

We can do *algebra* with *Differentials* !

Momentum & Impulse

$$F = m\,a \quad \& \quad a \equiv dv/dt \;\Rightarrow\; \boxed{m\,dv = F\,dt}$$

$$a \equiv dv/dt \quad \& \quad v \equiv dx/dt \;\Rightarrow\; v\,dv/dt = a\,dx/dt$$

Kinetic Energy & Work

Cancel dt's & add $F = m\,a \;\Rightarrow\; \boxed{mv\,dv = F\,dx}$

Antiderivatives: just ask,

"What Function Has This Derivative?"

if $g(x) = 2\,a\,x \; = df/dx$, what is $f(x)$?

Answer: $f(x) = \int 2\,a\,x\,dx \; = a\,x^2 \; + \mathrm{const.}$

Try this: if $g(x) = x^{-1} \; = df/dx$, what is $f(x)$?

So What?

Change in Momentum $p = m\,v$

= Impulse $\int F(t)\,dt$

(Useful when we know the *force* as a function of *time*.)

Change in Kinetic Energy $K = \tfrac{1}{2}\,m\,v^2$

= Work $\int F(x)\,dx$

(Useful when we know the *force* as a function of *position*.)

Finis